Odds and Chances

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One number that arose in my previous article really jumped out at me when I saw it. That was the fact that if you flop an inside straight draw, then the odds against hitting the straight by the river are about 5.1-to-1. Anyone who thinks about the odds in various situations probably uses 11-to-1 as the rough approximation of the odds against making an inside straight draw on each of the turn and the river as individual decisions. The fact that the odds against drop so much for the two-stage decision problem is something most people would notice. This is the motivation for this month's article.

Odds and probability are inexorably woven together. This results from the way in which they are defined. Let me use the word "event" to describe a particular result of some action we are about to perform. Examples would be being dealt a pair of aces in hold'em, having a spade appear as the turn card, rolling an immediate winner on the come out roll in craps, and so on. Furthermore, let's use the word "success" to mean an outcome of the action that produces the desired event, and "failure" to mean an outcome of the action that does not produce the event.

If A is the number of successful outcomes, B is the number of failed outcomes, and N is the total number of possible outcomes, then the first obvious fact to note is that $A + B = N$. An important consequence of this fact is that if we know any two of the three values, then we automatically know the third via the preceding equation. That, in turn, allows one to move back and forth easily between probability and odds.

The definitions are now straightforward. The probability of an event is defined to be the rational number A/N (we are assuming all the outcomes are equally likely), the odds against the event are B-to-A, and the odds for the event are A-to-B. Let's illustrate with a simple example.

Let the event be drawing a heart when randomly drawing a single card from a standard 52-card deck. There are 13 successes and 39 failures, that is, $A = 13$, $B = 39$, and $N = 52$. The probability of drawing a heart is then $13/52 = 1/4$. The odds against drawing a heart are 39-to-13 or 3-to-1. Finally, the odds for drawing a heart are 13-to-39 or 1-to-3.

As far as going back and forth between odds and probability is concerned, let's look at another example. Suppose I tell you the probability of some event is $2/5$ and I want you to tell me the odds against the event. From the given probability we know that proportionately $A = 2$ and $N = 5$. From the equation above we then know that $B = 3$. Therefore, the odds against the event are

3-to-2. Don't try to tell me this is difficult.

Those of you who know me are aware of a certain fussiness about proper language. The primary reason I believe proper language is important is that subtlety is difficult, if not impossible, to achieve in a landscape of improper English. When discussing odds, the correct terms are "odds against" and "odds for". When people approach me with a question about the "odds of" some event, I usually give them both the odds against and the odds for because I don't know for certain what they are asking. In many cases they actually want the probability of the event so that their phrasing is a real misuse of the term odds. It would be much better to ask what the chances are of the event.

There are no fixed rules as to when we should use odds and when we should use probability. There are situations for which odds are best and there are situations for which probability is best. Tradition also plays a role. For example, a baseball player who gets a hit once every three at bats has a batting average of .333. If you were to tell a fan that the odds against him getting a hit are 2-to-1, you would most likely receive a strange look. Baseball fans are accustomed to and comfortable with the probabilistic approach to expressing a batter's prowess. Trying to use odds in that context would perplex most fans.

On the other hand, odds are very useful in the poker world because many decisions players make are done most easily in terms of odds rather than probability. The classic example is using pot odds to decide whether to call a bet. For example, if a player is contemplating making a $10 call with $100 in the pot, then the pot odds are 10-to-1. If the player needs some help to make her hand, the essential break even point for calling the bet is when the odds against making her hand are 10-to-1. Thus, if the odds against making her hand are 10-to-1 or better, she should call. This is overly simplistic because there are frequently other factors to be considered as well, but the point is whether players would prefer to remember that the break even point is odds of 10-to-1 against or a probability of 1/11. Using odds here seems more natural and somewhat easier.

The main valid criticism of working with odds is that they are computationally useless for sequential actions. Let's return to the example motivating this article. Suppose a player flops a straight draw for which four cards will fill the straight. Of the 47 unseen cards, there are four successes and 43 failures. Thus, the odds against her making the straight on the turn are 43-to-4 which we approximate as 11-to-1. If she misses on the turn, then the odds against making the straight on the river are 42-to-4 which reduces to 21-to-2. Many people approximate this as 11-to-1 as well.

If someone now asks you what the odds against are for making a straight by the river, how do you get 5.1-to-1 using 43-to-4 and 21-to-2? The answer is "You don't." Instead, you need to work with the corresponding probabilities. The probability of missing a straight on the turn is 43/47, and the probability of missing the straight on the river, given that it is missed on the turn, is 21/23. To get the probability that the straight is missed on both the turn and the river, you simply multiply the two probabilities and get 903/1081. The preceding expression is a probability so that for the problem of whether we hit a straight by the river, we can treat it as 903 failures leaving 178 successes. This means the odds against are 903-to-178 which is approximately 5.1-to-1.