Starting Methods for High-order Two-step Runge–Kutta Methods

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Abstract.

In an attempt to reduce the large number of derivatives required in each step of an explicit Runge–Kutta method, two-step Runge–Kutta (TSRK) methods were formulated to exploit derivative evaluations from two steps for each new gridpoint approximation. Jackiewicz and Verner [2] solved appropriate order conditions directly to derive TSRK pairs of orders up to 7,8. These pairs were coded in a variable-step implementation; however, results from the selected pair of orders 7 and 8 manifested a lower order than expected. Results from some experiments suggested by Philip Sharp, identified the deficiency as a failure of values in the starting step to be suitably "perturbed" to match the error profile of off-step values in subsequent steps. This property has been previously reported by Hairer and Wanner [1].

In the talk, results of the experiments will clearly identify deficiencies of previous implementations. A suitable analysis leads to formulas which would correct the problem, and a derivation of special "starting" methods for TSRK methods will be presented. Finally, tables from a fixed stepsize implementation of modified methods will illustrate that the desired behaviour is achieved.

References

[1] E. Hairer and G. Wanner, *Order conditions for general two-step Runge–Kutta methods.*, SIAM J. Numer. Anal. **34** (1997), pp. 227–248.

[2] Z. Jackiewicz and J.H. Verner, *Derivation and implementation of two-step Runge–Kutta pairs*, Japan JIAM **19** (2002), pp. 227–248.

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