Strong stability preserving explicit Runge–Kutta methods of maximal effective order

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Abstract

We apply the concept of effective order to strong stability preserving (SSP) explicit Runge–Kutta methods. Relative to classical Runge–Kutta methods, methods with an effective order of accuracy are designed to satisfy a relaxed set of order conditions, but yield higher order accuracy when composed with special starting and stopping methods. We show that this allows the construction of four-stage SSP methods with effective order four (such methods cannot have classical order four). However, we also prove that effective order five methods—like classical order five methods—require the use of non-positive weights and so cannot be SSP. By numerical optimization, we construct explicit SSP Runge–Kutta methods up to effective order four and establish the optimality of many of them. Numerical experiments demonstrate the validity of these methods in practice.

1 Introduction

Strong stability preserving time discretization methods were originally developed for the solution of nonlinear hyperbolic partial differential equations (PDEs). Solutions of such PDEs may contain discontinuities even when the initial conditions are smooth. Many numerical methods for their solution are based on a method-of-lines approach in which the problem is first discretized in space to yield a system of ODEs. The spatial discretization is often chosen to ensure the solution is total variation diminishing (TVD), in order to avoid the appearance of spurious oscillations near discontinuities, *when coupled with first-order forward Euler time integration.* Strong stability preserving (SSP) time discretizations (also known as TVD discretizations [11]) are high-order time discretizations that guarantee the TVD property (or other convex functional bounds), with a possibly different step-size restriction [9]. Section 2 reviews Runge–Kutta methods and the concept of strong stability preserving methods.

Explicit SSP Runge–Kutta methods cannot have order greater than four [24]. However, a Runge–Kutta method may achieve an effective order of accuracy higher than its classical order by the use of special starting and stopping procedures. The conditions for a method to have effective order q are in general less restrictive than the conditions for a method to have classical order q. Section 3 presents

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