
Order Tests and Derivation of Two-Step Runge-Kutta Pairs of Order 8

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Contributed talk

In an attempt to improve the efficiency of explicit Runge–Kutta pairs, a number of attempts to construct related explicit general linear methods have appeared in the literature. Among these, some two-step Runge–Kutta pairs (TSRK) have shown promise in accurately estimating the local truncation error. Such methods of low stage-order must be started with a special procedure, and some Runge–Kutta "starting" methods for TSRK methods of order 6 are known to yield the design order.

For pairs of orders 7 and 8, some coefficients have been obtained but so far the design order appears not to be achieved. We have implemented algorithms for determining order conditions constructed by Butcher and Tracogna (*Applied Num. Math* 24, 351-364) to obtain an almost automatic numerical verification of order conditions up to order 9. These validate some TSRK pairs of orders 3,4 and 5,6, but some order conditions of order 8 for previously constructed methods of orders 7,8 are invalid. This paper contrasts order conditions appearing explicitly in Butcher and Tracogna with those in Jackiewicz and Tracogna (*SIAM J. NA* 32, 1390-1427) to identify some deficiencies in the order conditions solved by Jackiewicz and Verner (*JJIAM* 19, 227-248). Attempts to modify the order conditions solved to obtain methods which achieve order 8 are in progress. We may also hope that such a modification will also lead to the derivation of corresponding starting methods so that TSRK pairs of orders 7,8 may be implemented for contrast to explicit Runge–Kutta pairs of the same orders.