Math 303, Fall 2011, Lecture 13

(1) The Liar's paradox

This sentence is false.

If it is true then (according to itself) it is false

If it is false then (according to itself) it is false that it is false, that is it is true.

We can find lots of examples of presentations of this paradox

Fred only said two things in this comic

"What the heck are you doing?"

and

"I guess today's comic goes in the second pile, huh?"

strictly speaking those are both questions not statements

but the second one is really a statement framed as a question

"I claim this comic goes in the second pile"

give what the second pile is. This statement is

"This statement is false"

This one's not quite the liar's paradox as Wally isn't saying any specific statement is false - merely implying it.

Self reference (a sentence talking about itself) isn't necessary for the paradox

A: Sentence B is true
B: Sentence A is false

) equivalent to what is on the card

A true ⇒ B true ⇒ A false ⇒ B false

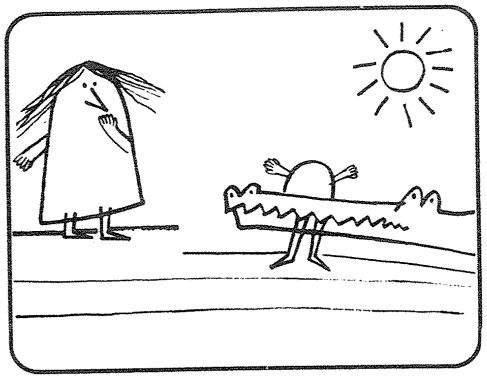
A colleague of mine says he has a foolproof pick up line

"If I were to ask you out would you give the same answer as the answer you'll give for this question"

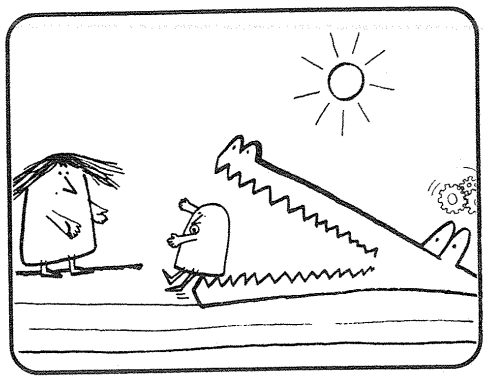
Answer to blue question	Answer to will you go out with me
yes	yes
no	yes

It can't fail!

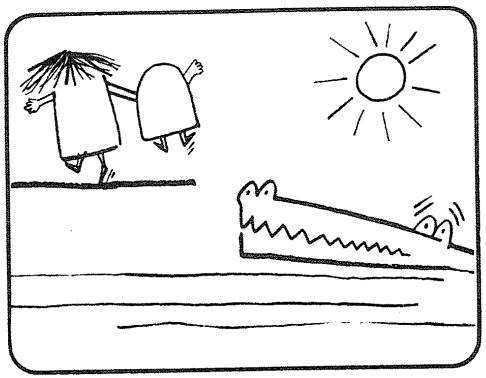
Crocodile and Baby



Greek philosophers liked to tell about a crocodile that snatched a baby from its mother.
Crocodile: Will I eat your baby? Answer correctly and I'll give the baby back to you unharmed.
Mother: Oh! Oh! You're going to eat my baby.



Crocodile: Hmmm. What shall I do? If I give you back your baby, you will have spoken falsely. I should have eaten it. . . . Okay, so I won't give it back.
Mother: But you must. If you eat my baby, I spoke correctly and you have to give it back.



The poor crocodile was so freaked that it let the baby go. The mother grabbed her child and ran.
Crocodile: Zounds! If only she'd said I'd give the baby back. I'd have had a juicy meal.

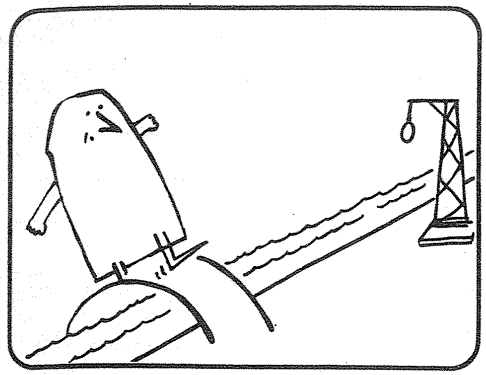
The crocodile has a problem. He has to both eat the baby and give it back, at the same time.

The mother is very clever. Suppose, instead, she had said: "You're going to give the baby back." Then, the crocodile could return the baby or eat it, in both cases without contradiction. If he gives it back, the mother spoke truly, and the crocodile has kept his word. On the other hand, if he is mean enough, he can eat the baby. This makes the mother's statement false, which frees the crocodile from the obligation to give the baby back.

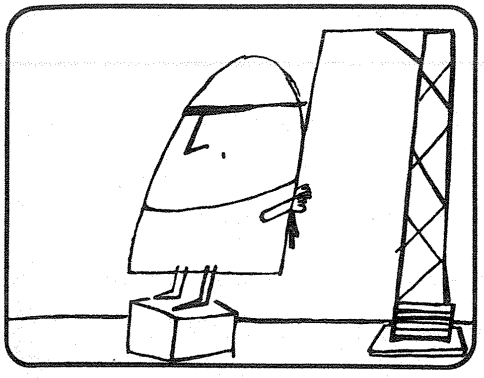
Mother's answer	Crocodile's action	Paradox ?
not eat	eat	no
not eat	not eat	no
eat	eat	yes
eat	not eat	yes

From Aha Gotcha by Martin Gardner

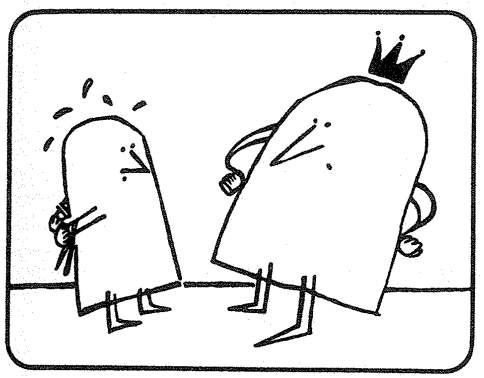
The Don Quixote Paradox



The novel *Don Quixote* tells of an island with a curious law. A guard questions every visitor:
Guard: Why are you coming here?
If the visitor answers truly, all is well. If he answers falsely he is hanged.



One day a visitor answered:
Visitor: I came here to be hanged!
The guards were as puzzled as the crocodile. If they do not hang the man, he has lied and has to hang. But if they hang him, he spoke truly and should not be hanged.



To decide the matter, the visitor was taken to the island's governor. After thinking long and hard the governor made his decision.
Governor: Whatever I decide is sure to break the law. So I will be merciful and let the man go free.

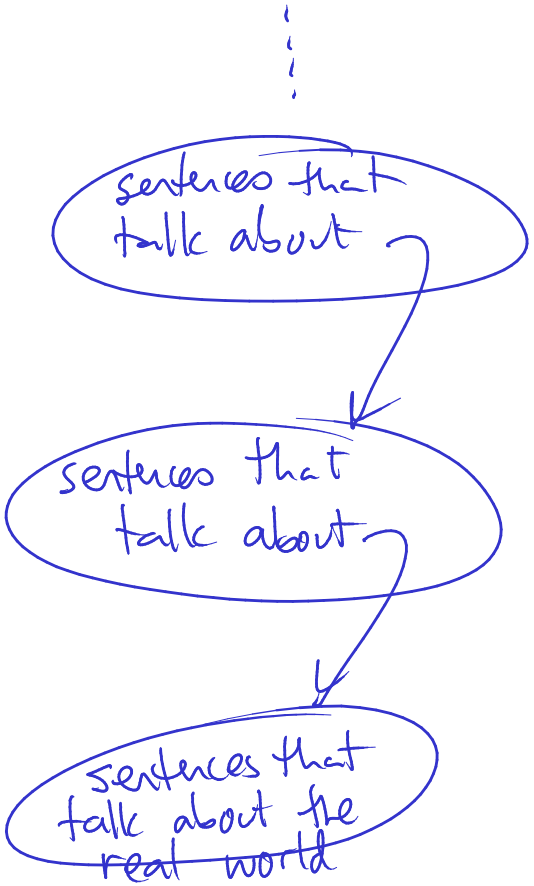
The hanging paradox is in Chapter 51 of the second book of *Don Quixote*. Sancho Panza, the Don's servant, has become governor of an island where he has sworn to uphold the country's curious law about visitors. When the visitor is brought before him, he decides the man's case with mercy and common sense.

The paradox, although similar to the crocodile paradox, is clouded by the ambiguity of the visitor's statement. Is it the man's statement about his intent, or is it a statement about a future event? In the first sense, the man may have spoken truly about his intent, and the authorities could then not hang him and there would be no contradiction. But if his statement is taken in the second sense, then whatever the authorities do will contradict the law.

From Aha Gotcha by Martin Gardner

How do we resolve the liar's paradox?

Tarski's Metalanguage



Or maybe there's some space between true and false
or maybe a sentence can be in an indetermined state

Gödel's first incompleteness theorem says that if a formal theory is

- strong enough to express elementary arithmetic
- consistent

then there are true statements about arithmetic which are not provable within the theory

He shows this by expressing the statement

This statement is unprovable

in arithmetic in the formal theory

(2) After the midterm.

- Predicate calculus (the rest of the rules for our system)
- The axioms of set theory expressed in first order logic

Please read Cohen ch II section 1