MATH 817 ASSIGNMENT 3

DUE OCTOBER 22, 2009, IN CLASS

If your assignment must be late for any reason please notify me (by email, phone or in person) **before** the assignment is due. There will be no retroactive lates.

(1) Let $\phi : G \to H$ be a surjective homomorphism. Show $\phi(G^n) = H^n$.

(2) (Isaacs problem 8.10). A group G is *supersolvable* if there exist normal subgroups N_i with

$$1 = N_0 \subseteq N_1 \subseteq \cdots \subseteq N_n = G$$

and such that N_{i+1}/N_i is cyclic for $0 \leq i < n$. Show that a finite nilpotent group is necessarily supersolvable.

(3) (Isaacs problem 8.16). Show that a minimal normal subgroup of a supsersolvable group is cyclic.

(4) (Isaacs problem 2.12c, Lemma 8.26) For $x, y, z \in G$ let $[x, y, z] = [[x, y], z]$. Prove

$$[x, y^{-1}, z]^y [y, z^{-1}, x]^z [z, x^{-1}, y]^x = 1$$

(5) (a) Show that every group of order 4 is solvable.
(b) Show that every group of order pq, p and q distinct primes, is solvable.
(c) Show that every group of order 12 is solvable.
(d) Show that every group of order 36 is solvable.
Hint, consider Sylow subgroups and note that if $H \lhd G$, $|G : H| = n$ then there is a homomorphism $G \to S_n$.