MATH 817 ASSIGNMENT 6

DUE DECEMBER 3, 2009, IN CLASS

If your assignment must be late for any reason please notify me (by email, phone or in person) **before** the assignment is due. There will be no retroactive lates.

(1) (Isaacs problem 13.10) Let R be a ring and $e \in R$ an idempotent. Show that $J(eRe) = eJ(R)e$.

(2) A right ideal I of R is *small* if for every right ideal K
$$K + I = R \implies K = R.$$
Prove that I is small iff $I \subseteq J(R)$.

(3) Let G be a group and $\mathbb{C}[G]$ its group ring over $\mathbb{C}$.
   (a) Show that every element of the form $\lambda - a$, $\lambda \in \mathbb{C} \smallsetminus \{0\}$, $a \in J(\mathbb{C}[G])$, is a unit in $\mathbb{C}[G]$.
   (b) Suppose G is countable. For fixed $a \in J(\mathbb{C}[G])$, show that
   $$\mathcal{S} = \left\{ \frac{1}{\lambda - a} : \lambda \in \mathbb{C} \smallsetminus \{0\} \right\}$$
   is linearly dependent over $\mathbb{C}$.
   (c) Suppose G is countable. Show that every element of $J(\mathbb{C}[G])$ is algebraic over $\mathbb{C}$.
   (d) Suppose G is countable. Show that $J(\mathbb{C}[G])$ is nil.
   (e) Suppose G is any group. Show that $J(\mathbb{C}[G])$ is nil.
   (f) For $\alpha = \sum_{i=1}^{n} \lambda_i g_i \in \mathbb{C}[G]$ define $\alpha^* = \sum_{i=1}^{n} \bar{\lambda}_i g_i^{-1}$, where $\bar{}$ denotes complex conjugation. If $0 \neq \alpha \in J(\mathbb{C}[G])$ show that $\alpha \alpha^*$ is not nilpotent.
   (g) Conclude (Rickart, Amitsur) that for any group $J(\mathbb{C}[G]) = 0$.

(4) Describe the structure of $\mathbb{C}[S_3]$.

(5) Calculate the character table of D_{12}.