MATH 821, SPRING 2013, ASSIGNMENT 4

(1) Show, by finding a bijection, that the number of self conjugate partitions of n is the same as the number of partitions with odd parts and no repeated parts.

(2) Show that QED is renormalizable in the sense that the superficial degree of divergence of a graph depends only on the external edges of the graph.

(3) Represent a semistandard Young tableau of shape λ with m rows as m lattice paths as follows: Let n be any number larger than the largest entry of the tableu. Start the ith path at $P_i = (m - i, 1)$ and end at $Q_i = (\lambda_i + m - i, n)$, read across row i, for each symbol j in row i put a horizontal step at $y = j$ in the ith path and then put vertical steps to reach the height of the next entry in the row. For example given the tableau

$$
\begin{array}{ccccc}
1 & 3 & 3 & 3 & 6 \\
4 & 4 & 5 & & \\
6 & 7 & & &
\end{array}
$$

we can take $n = 8$ and get the paths
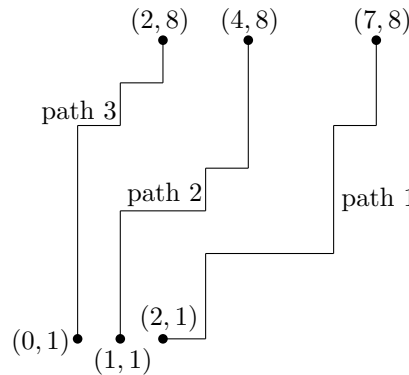
Show that for any semistandard Young tableau these paths are always non-intersecting and that every non-intersecting m-tuple of paths using left and up steps from the Ps to the Qs gives a semistandard Young tableau of shape λ. *This connection to paths is one way to get at a whole family of determinantal identities for symmetric functions.*

(4) Find 3 typos in the scribed notes. Let the person who made those notes know about the typos you found and correct any typos your colleagues tell you about in your notes. For this question you should hand in the three typos you found with your assignment and you should email me corrected notes if your colleagues found any errors in yours.