faculty of science
department of mathematics

Lecture 01W

math 322
muraki

Note Title 05/01/2007

A) ONLINE MATERIALS

- webct
- course webpage → 1st week only + emergency back-up

SYLLABUS:

STUDENT INFO FORM + PRE-REQ SELF-EVALUATION

HOMEWORK #0 → due WED 12 SEPT. + INFO FORM
GUIDE TO WRITTEN WORK

HOMEWORK #-1 → optional

PRE-CLASS NOTES: posted evening prior to lectures

B) MATH 322 CLASSROOM ELECTRONICS

To minimize classroom distractions from electronic devices, the following policy will be applied:

1) All course members are expected to respect the audio & visual environment of their classmates.

2) No electronic devices may be in use (ie zero tolerance) in the designated "e-free zone" (for c9000, this is all seats except for the back 3 rows) --- except by those students who have a contract for specific educational purposes.

3) Students who are permitted the use of electronic devices have agreed to use them in a manner that minimizes the distraction to others. Violation of this agreement invalidates their permission. (E-contracts are to be arranged with the instructor.)

The Cornell Note-taking System

2 1/2”

6”

Notetaking Column

Cue Column

1. **Record**: During the lecture, use the notetaking column to record the lecture using telegraphic sentences.
2. **Questions**: As soon after class as possible, formulate questions based on the notes in the right-hand column. Writing questions helps to clarify meanings, reveal relationships, establish continuity, and strengthen memory. Also, the writing of questions sets up a perfect stage for exam-studying later.
3. **Recite**: Cover the notetaking column with a sheet of paper. Then, looking at the questions or cue-words in the question and cue column only, say aloud, in your own words, the answers to the questions, facts, or ideas indicated by the cue-words.
4. **Reflect**: Reflect on the material by asking yourself questions, for example: “What’s the significance of these facts? What principle are they based on? How can I apply them? How do they fit in with what I already know? What’s beyond them?
5. **Review**: Spend at least ten minutes every week reviewing all your previous notes. If you do, you’ll retain a great deal for current use, as well as, for the exam.

Summary

2”

After class, use this space at the bottom of each page to summarize the notes on that page.

Adapted from How to Study in College 7/e by Walter Pauk, 2001 Houghton Mifflin Company

c) WHAT IS "COMPLEX VARIABLES" ALL ABOUT ?

- a calculus of functions based on the arithmetic of complex numbers & the imaginary quantity $i = \sqrt{-1}$

- complex-valued functions with derivatives are "very special" & have magical (!) properties

D) PREPAREDNESS FOR MATH 322

- mathematics classes on education spectrum

knowledge-based (facts) $\longrightarrow$ skills-based (ideas)

- MATH learning strategies much like LANGUAGE courses

calc 1	basic vocabulary
calc 2	simple phrases & sentences
calc 3	compound sentences
math 322	paragraph construction

- calc 3 (math 251) is essential pre-requisite (self-eval)

E) WHAT IS MATH 322 ABOUT ?

- calculus 1, 2 & 3 are calculationally-intensive
- complex analysis is a calculus that RELIES on more abstract understanding.

- greater emphasis on proofs & analysis

  logical deduction

  - yet calculations still play a large role
  - transition class to MATH 320, real analysis
  - background math for physics & signal processing

$\simeq 1/3$ of class are math

F) ASSIGNMENTS

- graded on quality of presentation

  CORRECTNESS , CLARITY , CONCISENESS

- homework problems styled on exam-type of questions
  - emphasis on IDEAS behind calculations

- less "WHAT IS THE RIGHT ANSWER?", rather "WHY IS IT THE RIGHT ANSWER?"
- WHY demands some words of explanation

- exam problems will not be duplicates of assigned problems
- optional textbook problems for practice on key calculations

E) DEFINITION OF A COMPLEX NUMBER (§1.1)

- sect 1.1-8 of text, review of complex-valued arithmetic
  - see also Appendix H of Stewart's Calculus

- the imaginary number, i is defined to be a square root of -1, so that $i^2 =$

{ in engineering, j is used as $\sqrt{-1}$ }

- a complex number is defined by $z = x + iy$ where x & y are numbers

  - the imaginary number i is represented by $x =$, $y =$
  - for example, $z = i =$ $+$ i.
  - is called the real part of z: $=$ _ (z)
  - " " " imaginary part : $=$ _ (z)

- text adds an ordered pair notation (p1)
  $z = x + iy \equiv$ x y

• first i, EULER 1794 (presented 1777)

semper per logarithmos et arcus circulares integrari posse, id quod a casibus simplicioribus inchoando in sequentibus problematibus ostendere constitui.

Problema 1.

§. 2. *Proposita formula differentiali* $\frac{\partial \Phi \cos. \Phi}{\sqrt[n]{\cos. n\Phi}}$, *ejus integrale per logarithmos et arcus circulares investigare.*

Solutio.

Quoniam mihi quidem alia adhuc via non patet istud praestandi, nisi per imaginaria procedendo, formulam $\sqrt{-1}$ littera i in posterum designabo, ita ut sit $ii = -1$, ideoque $\frac{1}{i} = -i$. Jam ante omnia in numeratore nostrae formulae loco $\cos. \Phi$ has duas partes substituamus

$$\tfrac{1}{2}(\cos. \Phi + i \sin. \Phi) + \tfrac{1}{2}(\cos. \Phi - i \sin. \Phi),$$

atque ipsam formulam propositam per duas hujusmodi partes repraesentemus, quae sint

$$\partial p = \frac{\partial \Phi (\cos. \Phi + i \sin. \Phi)}{\sqrt[n]{\cos. n\Phi}} \text{ et } \partial q = \frac{\partial \Phi (\cos. \Phi - i \sin. \Phi)}{\sqrt[n]{\cos. n\Phi}},$$

ita ut ipsa formula nostra proposita sit $\frac{1}{2}\partial p + \frac{1}{2}\partial q$, ideoque ejus integrale $\frac{p+q}{2}$.

§. 3. Nunc ambas istas partes seorsim sequenti modo tractemus. Pro formula scilicet priore

$$\partial p = \frac{\partial \Phi (\cos. \Phi + i \sin. \Phi)}{\sqrt[n]{\cos. n\Phi}} \text{ statuamus } \frac{\cos. \Phi + i \sin. \Phi}{\sqrt[n]{\cos. n\Phi}} = x,$$

ut sit $\partial p = x\partial\Phi$, ac sumtis potestatibus exponentis n habebimus

• i forever, Gauss 1801 (disquisitiones arithmeticae)

Aequationes pro functionibus trigonometricis arcuum, qui sunt pars aut partes totius peripheriae: reductio functionum trigonometricarum ad radices aequationis $x^n - 1 = 0$.

337.

Satis constat, functiones trigonometricas omnium angulorum $\frac{kP}{n}$, denotando per k indefinite omnes numeros $0, 1, 2 \ldots n-1$, per radices aequationum n^{ti} gradus exprimi, puta *sinus* per radices huius (I)

$$x^n - \frac{1}{4} n x^{n-2} + \frac{1}{16} \frac{n \,.\, n-3}{1 \,.\, 2} x^{n-4} - \frac{1}{64} \frac{n \,.\, n-4 \,.\, n-5}{1 \,.\, 2 \,.\, 3} x^{n-6} + \text{etc.} \pm \frac{1}{2^{n-1}} n x = 0$$

cosinus per radices huius (II)

$$x^n - \frac{1}{4} n x^{n-2} + \frac{1}{16} \frac{n \,.\, n-3}{1 \,.\, 2} x^{n-4} - \frac{1}{64} \frac{n \,.\, n-4 \,.\, n-5}{1 \,.\, 2 \,.\, 3} x^{n-6} + \text{etc.} \pm \frac{1}{2^{n-1}} n x - \frac{1}{2^{n-1}} = 0$$

denique *tangentes* per radices huius (III)

$$x^n - \frac{n \,.\, n-1}{1 \,.\, 2} x^{n-2} + \frac{n \,.\, n-1 \,.\, n-2 \,.\, n-3}{1 \,.\, 2 \,.\, 3 \,.\, 4} x^{n-4} - \text{etc.} \pm n x = 0$$

Hae aequationes (quae generaliter pro quovis valore impari ipsius n valent, II vero pro pari quoque), ponendo $n = 2m+1$, facile ad gradum m^{tum} deprimuntur; scilicet I et III, dividendo partem a laeva per x et substituendo y pro xx. Aequatio II autem manifesto radicem $x = 1$ $(= \cos 0)$ implicat, et e reliquis binae semper aequales sunt $(\cos \frac{P}{n} = \cos \frac{(n-1)P}{n}$, $\cos \frac{2P}{n} = \cos \frac{(n-2)P}{n}$ etc.); quare ipsius pars a laeva per $x - 1$ divisibilis, quotiensque quadratum erit, cuius radicem quadratam extrahendo, aequatio II reducitur ad hanc

$$x^m + \frac{1}{2} x^{m-1} - \frac{1}{4} (m-1) x^{m-2} - \frac{1}{8} (m-2) x^{m-3}$$
$$+ \frac{1}{16} \frac{m-2 \,.\, m-3}{1 \,.\, 2} x^{m-4} + \frac{1}{32} \frac{m-3 \,.\, m-4}{1 \,.\, 2} x^{m-5} - \text{etc.} = 0$$

cuius radices erunt cosinus angulorum $\frac{P}{n}, \frac{2P}{n}, \frac{3P}{n} \ldots \frac{mP}{n}$. Ulteriores reductiones harum aequationum, pro eo quidem casu, ubi n est numerus primus, hactenus non habebantur.

Attamen nulla harum aequationum tam tractabilis et ad institutum nostrum tam idonea est, quam haec $x^n - 1 = 0$, cuius radices cum radicibus illarum arctissime connexas esse constat. Scilicet, scribendo brevitatis caussa i pro quantitate imaginaria $\sqrt{-1}$, radices aequationis $x^n - 1 = 0$ exhibentur per

$$\cos \frac{kP}{n} + i \sin \frac{kP}{n} = r.$$

- number sets

denotes the set of all numbers

- the set of all numbers

$= \{ \; = \; + i \; | \; , \; \in \; \}$

4) WHY DO WE WANT $\mathbb{C}$?

- zeros of a quadratic polynomial

$p(z) = \quad + \quad + \quad =$

(a, b, c are real #'s)

- quadratic formula

$$z_{\pm} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a) roots satisfy $\quad p(z_{\pm}) =$

b) sum of roots $\quad z_+ + z_- = -$

c) product of roots $\quad z_+ \cdot z_- =$

- Cardano's (1501-76) example

  $z(10-z) = 40 \rightarrow a=1,\ b=-10,\ c=40$

  - quad formula gives

    $$z_\pm \overset{!}{=} \frac{\quad \pm \sqrt{\quad}}{\quad}$$

  - if we think of this expression as

    $$z_\pm \overset{?}{=} \quad \pm \frac{\sqrt{\quad}}{\quad}$$

    $$\overset{?}{=} \quad \pm \sqrt{\quad} \cdot \sqrt{\quad} \overset{?}{=} \underbrace{\quad}_{x} \pm \underbrace{i \sqrt{\quad}}_{y}$$

  - is this a useful way to think?
  - can we check $z_+ + z_- \overset{?}{=} -(\quad)$

    $z_+ \cdot z_- \overset{?}{=}$

NEED TO DEFINE A <u>SENSIBLE</u> ARITHMETIC

I) COMPLEX ARITHMETIC OPERATIONS (§1.2 + 1.3)

- addition, z_1 z_2

$$(x_1 + iy_1) \quad (x_2 + iy_2) \equiv (\underbrace{\qquad}_{\text{real}}) + i(\underbrace{\qquad}_{\text{real}})$$

- multiplication, $z_1 \cdot z_2$, definition based on distrib. + comm. laws for $\mathbb{R}$

$$(x_1 + iy_1) \cdot (x_2 + iy_2) \equiv \quad + \quad i \quad + i \quad + i \quad i$$

$$\equiv (\underbrace{x_1 x_2 - y_1 y_2}_{\text{real}}) + i(\underbrace{x_1 y_2 + x_2 y_1}_{\text{real}})$$

- through the above definitions, -valued arithmetic inherits the familiar __________ (p 3)

  - commutative $\quad z_1 + z_2 = z + z \quad + \quad z_1 \cdot z_2 = z \cdot z$
  - associative $\quad (z_1 + z_2) + z_3 = z_1 + \quad z_2 + z_3$

    $(z_1 \cdot z_2) \cdot z_3 = z_1 \cdot \quad z_2 \cdot z_3$
  - distributive

$$z_1 \cdot (z_2 + z_3) = \quad \cdot \quad + \quad \cdot$$

- additive inverse $\qquad z + (\quad) = 0$

$$= (\quad) + i(\quad)$$

- multiplicative inverse $\qquad z(\quad) = 1 = 1 + i0$

$$= \left(\frac{x}{x^2+y^2}\right) - i\left(\frac{y}{x^2+y^2}\right) \qquad \text{for } z \neq$$

↳ p4, using 2×2 linear solve

also, $\qquad z_1 \cdot z_2 \neq \qquad$ if and only if $\qquad z_1 \neq$ and $z_2 \neq$

(p6)

- all of these arithmetic laws can be rigourously proved from the definitions of   &   and the knowledge of the   for   arith.

J) MORE ARITHMETIC

- magnitude / modulus of z (s1.4)

$$|z| \equiv \sqrt{\quad + \quad} \geq \qquad \text{AND} \qquad \text{-valued.}$$

- zero modulus property: $|z| = \quad$ only when $z =$

- complex conjugate (s1.5)

$$\bar{z} \text{ or } z^* \equiv x - iy$$

- eg $(3+4i)^* =$

$(1-i)^* =$

diff of squares

- this implies $z \cdot z^* = (x+iy)(x-iy)$

$$= \quad ^2 - (\quad)^2 = \qquad =$$

- multiplicative inverse

$$\frac{1}{z} = \frac{1 \cdot}{z \cdot} = \frac{1}{\quad} \cdot z$$

$$= \left(\frac{\quad}{x^2+y^2}\right) \quad i \left(\frac{\quad}{x^2+y^2}\right)$$

- Re + Im parts

$$\text{Re } z = \frac{\quad + \quad}{\quad} \quad ; \quad \text{Im } z = \frac{\quad - \quad}{\quad}$$

- more arithmetic identities

$$(z_1 + z_2)^* = z_1^* + z_2^*$$

$$(z_1 \cdot z_2)^* = z_1^* \cdot z_2^*$$

$$|z_1 \cdot z_2| = |z_1| \cdot |z_2|$$

- triangle inequality (p11)

$$|z_1 + z_2| \leq |z_1| + |z_2|$$

inequalities only apply to $\mathbb{R}$ numbers
($\mathbb{C}$-numbers CANNOT be ordered)

- 2-sided form (p11)

$$\big||z_1| - |z_2|\big| \leq |z_1 \pm z_2| \leq |z_1| + |z_2|$$

- key features of $\mathbb{C}$ arithmetic
  1) CLOSURE IN $\mathbb{C}$: results of arith are always $\mathbb{C}$ numbers (except div by zero)
  2) CONSISTENT WITH $\mathbb{R}$-VALUES ARITH.