APMA 900 • Advanced Mathematical Methods I • Fall 2011

Exact & Asymptotic Methods for Differential Equations

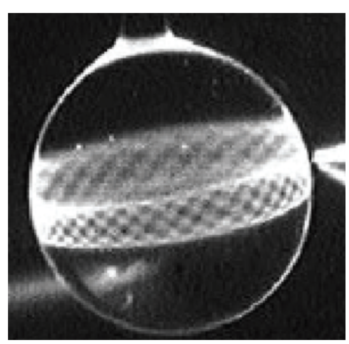
The course begins with an overview of classes of ordinary and partial differential equations (ODEs & PDEs) that are solved by exact methods. Fourier methods for solving linear DEs are extended to integral solution methods that include the Fourier and Laplace transforms. Investigation of this solution perspective establishes the close connection between complex variable theory and DEs. Other generalizations lead to the development of Sturm-Liouville eigenfunctions, function (Hilbert) spaces and special functions.

But many ODEs and PDEs encountered in applications are not amenable to exact solution. Nonetheless, a variety of so-called asymptotic methods are available for extracting analytical understanding. These approximate methods can also be useful in benchmarking numerically-computed solutions, and even decoding exact solutions whose formula complexity defies interpretation. Perturbation theory analyzes problems that are "nearby" to those with known exact properties. This perspective also gives mathematical insight into the consequences of approximations which neglect complicating effects in the development of model equations. Yet other types of asymptotic methods address singular situations where small changes to DE problems have a large impact on the solution. Examples include techniques known as: multiple-scale, averaging, WKB (geometrical optics) and boundary-layer methods.

Lectures will be based upon a case-study approach of ODE & PDE examples. Computational illustration will be an important tool for the lectures and assigned work. The rudiments of numerical computing will be developed through the use and modification of downloadable Matlab scripts.

Calendar course prerequisites: Undergraduate introduction to ODEs and linear PDEs. Other useful background includes real & complex analysis, elementary numerical analysis &/or scientific computing. (SFU undergraduates with Math 418 credit are welcome to request enrollment.)

Further information & updates: www.math.sfu.ca/∼muraki




These images are visual representations of basic PDE behaviours. A roll-up of clouds reveals an airflow instability caused by the Kelvin-Helmholtz mechanism. Laser light trapped as a whispering gallery mode in a 300-μm glass sphere illuminates the principles of wave propagation & refraction. The wake of a moving ship displays a geometric pattern governed by wave interference.