APMA 900 • Asymptotic Analysis of Differential Equations • Fall 2017

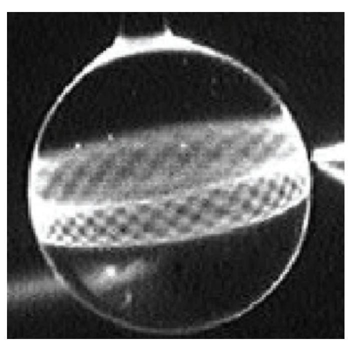
One aim of this course is to provide an overview of ordinary and partial differential equations (ODEs & PDEs) that are solved by exact methods. Fourier series methods for solving linear DEs are extended to integral solution methods that include the Fourier and Laplace transforms. Investigation of this solution perspective establishes the close connection between complex variable theory and DEs. Other generalizations lead to the development of Sturm-Liouville eigenfunctions, function (Hilbert) spaces and special function theory.

But many ODEs and PDEs encountered in applications are not amenable to exact solution. The other aim of this course is to introduce a variety of asymptotic methods that extend our analytical toolbox beyond exact theory. These approximate approaches can also be useful in benchmarking numerically-computed solutions, and even decoding exact solutions whose formula complexity defies interpretation. Perturbation theory analyzes problems that are "nearby" to those with known exact properties. This perspective also gives mathematical insight into the consequences of approximations which neglect complicating effects in the development of model equations. Yet other types of asymptotic methods address singular situations where small changes to DE problems have a large impact on the solution. Examples include techniques known as: multiple-scale, averaging, WKB (geometrical optics) and boundary-layer methods.

Lectures will be based upon a case-study approach of ODE & PDE examples. Computational illustration will be an important tool for the lectures and assigned work. Computer visualization and numerical computing will involve the use and modification of Matlab scripts.

Calendar course prerequisites: Undergraduate introduction to ODEs and linear PDEs. Other useful background includes real & complex analysis, elementary numerical analysis &/or scientific computing. (SFU undergraduates with Math 418 credit are encouraged to consider joining.)

Further information & updates: www.math.sfu.ca/∼muraki



These images are visual representations of some basic PDE behaviours. Instability, as visualized in an atmospheric airflow by the roll-up of a cloud-top. Wave refraction, as illuminated by laser light trapped as a whispering gallery mode in a 300-μm glass sphere. Wave interference, as apparent in the geometric pattern within the wake of a moving ship.