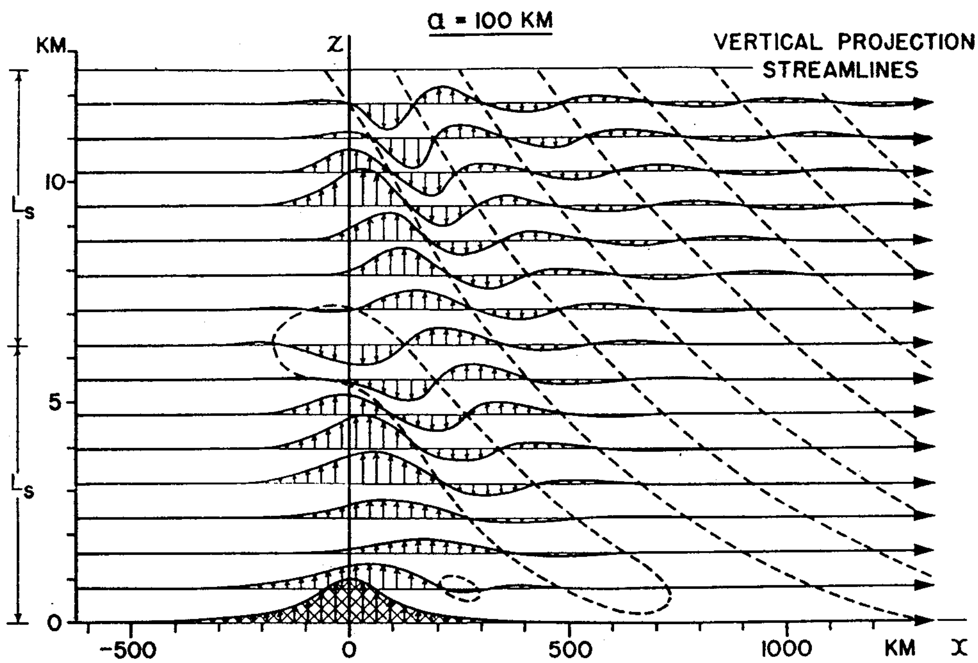
Revisiting Queney's Flow over a Mesoscale Ridge

stratified, hydrostatic & rotating



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Queney's Displacement Streamlines

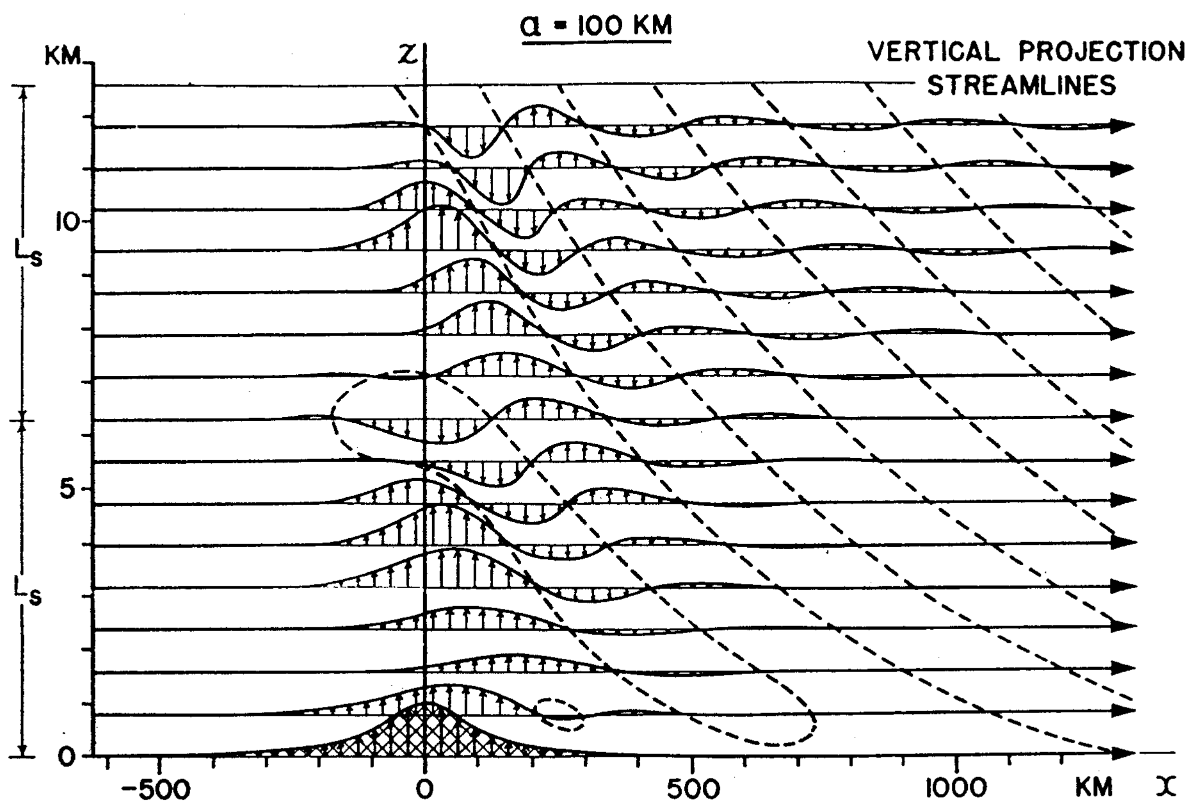
Flow over a 2D Mesoscale Ridge

- Queney 1947, 1948; Smith 1979; Gill 1982
- vertical displacement from buoyancy anomaly $b(x, z)$

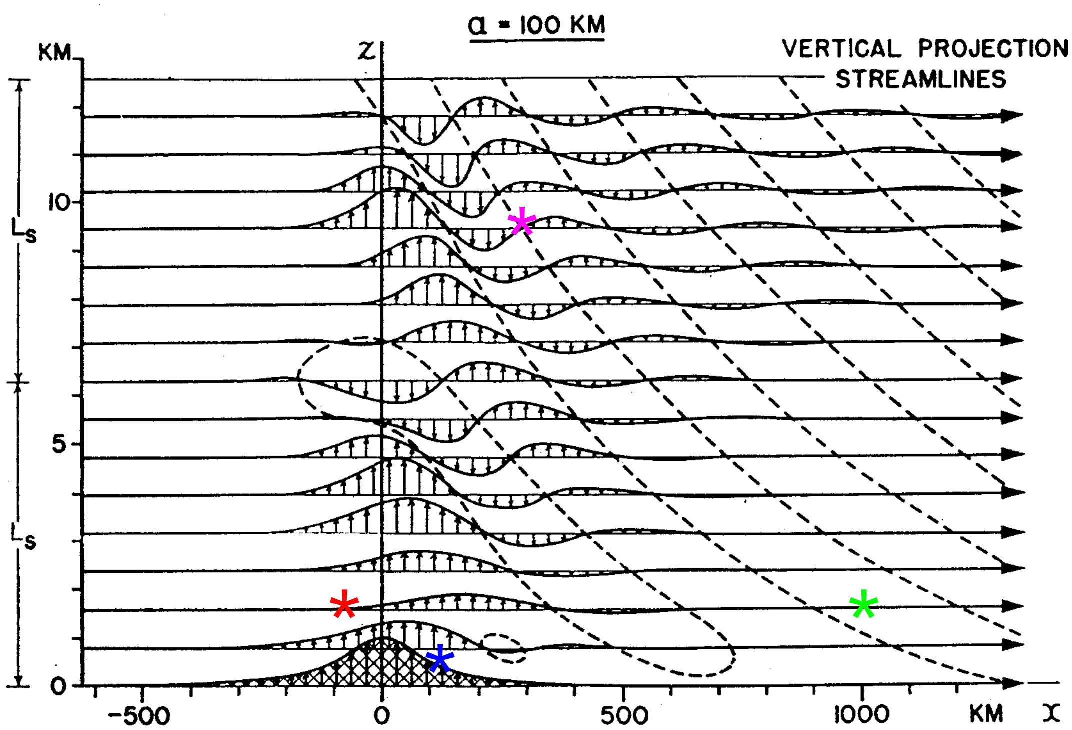
$$z(x) = z^{\infty} - \frac{1}{N^2}\, b(x, z^{\infty})$$

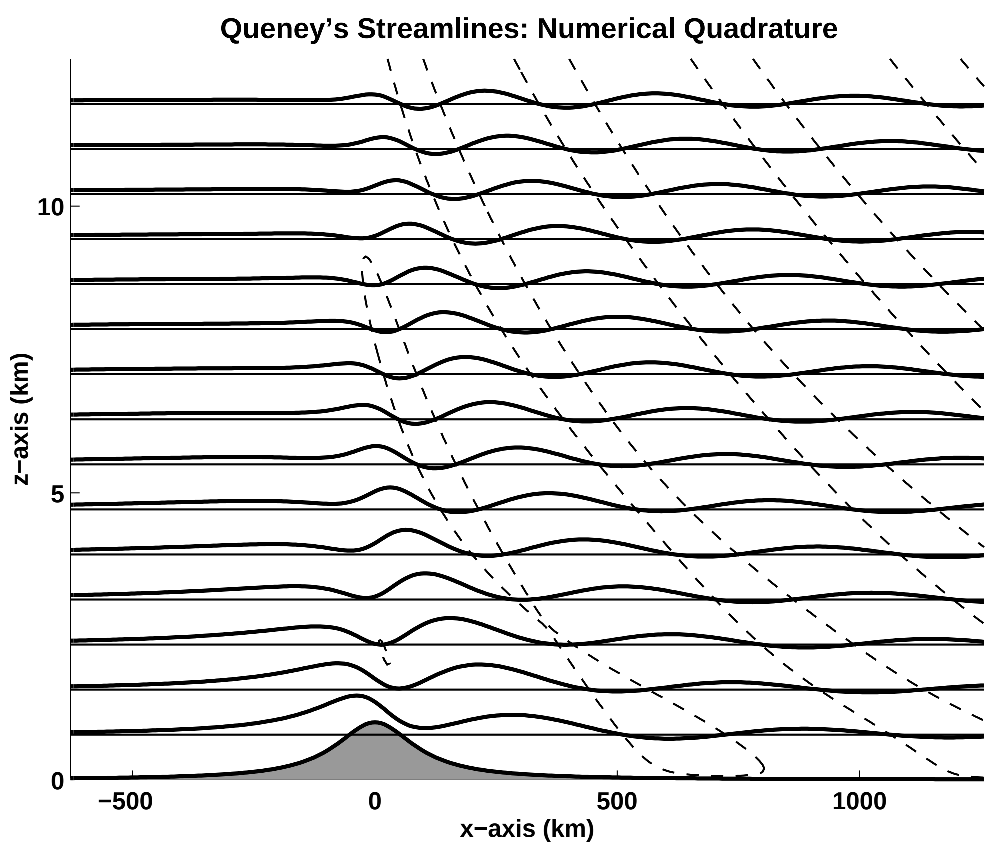
- rotating & hydrostatic case: parameters

$$\mathcal{R} = \frac{U}{fL} = 1 \qquad ; \qquad \mathcal{F} = \frac{U}{NH} = 1$$



Displacements Recomputed





Comparison

Missing Features in Queney 1947

- windward maxima of upward displacement (low level) *

  $\longrightarrow$ as in non-rotating case
- organized downdraft into downslope windstorm *
- convergence of (low level) streamlines in lee *

  $\longrightarrow$ as consistent with pressure drag in non-rotating case
- persistence of low level waves downstream *

  $\longrightarrow$ as surface analysis of (Pierrehumbert 1984)
- upward mean vertical displacement of far-field waves *

  $\longrightarrow$ as in QG theory

Two Fourier Calculations

- Queney's calculation: based on approximate analyses

  $\longrightarrow$ primarily *stationary phase* for far-field waves

  $\longrightarrow$ problematic at surface, summit & ridge zenith
- our direct quadrature of Fourier integral

  $\longrightarrow$ integrand has oscillatory singularity

  $\longrightarrow$ FFT-periodicity & severe aliasing issues

  $\longrightarrow$ we resolve using desingularized quadrature

Queney's Linear Theory (1947)

Rotating Case

- linear theory $\longrightarrow$ Fourier integral solution
- buoyancy anomaly

$$b(x, z) = -\frac{N^2}{\pi} \text{Real} \left\{ \int_0^\infty \hat{h}(k)\ e^{ikx}\ e^{im(k)z}\ dk \right\}$$

- inertial wavenumber (k_f) & Scorer parameter (k_s)

$$k_f = \frac{f}{U} \quad ; \quad k_s = \frac{N}{U}$$

incident wind U, f-plane Coriolis, stratification N

- 2D linear dispersion relation with rotation

$$m(k) = \begin{cases} i k_s \dfrac{k}{\sqrt{{k_f}^2 - k^2}} & \text{for} \quad 0 \le k < k_f \\ \\ k_s \dfrac{k}{\sqrt{k^2 - {k_f}^2}} & \text{for} \quad k_f < k < \infty \end{cases}$$

small k $\longrightarrow$ vertical decay; large k $\longrightarrow$ outgoing waves

- bell-shaped topography & Fourier transform

$$h(x) = \frac{HL^2}{L^2 + x^2} \quad ; \quad \hat{h}(k) = \pi H L\ e^{-|k|L}$$

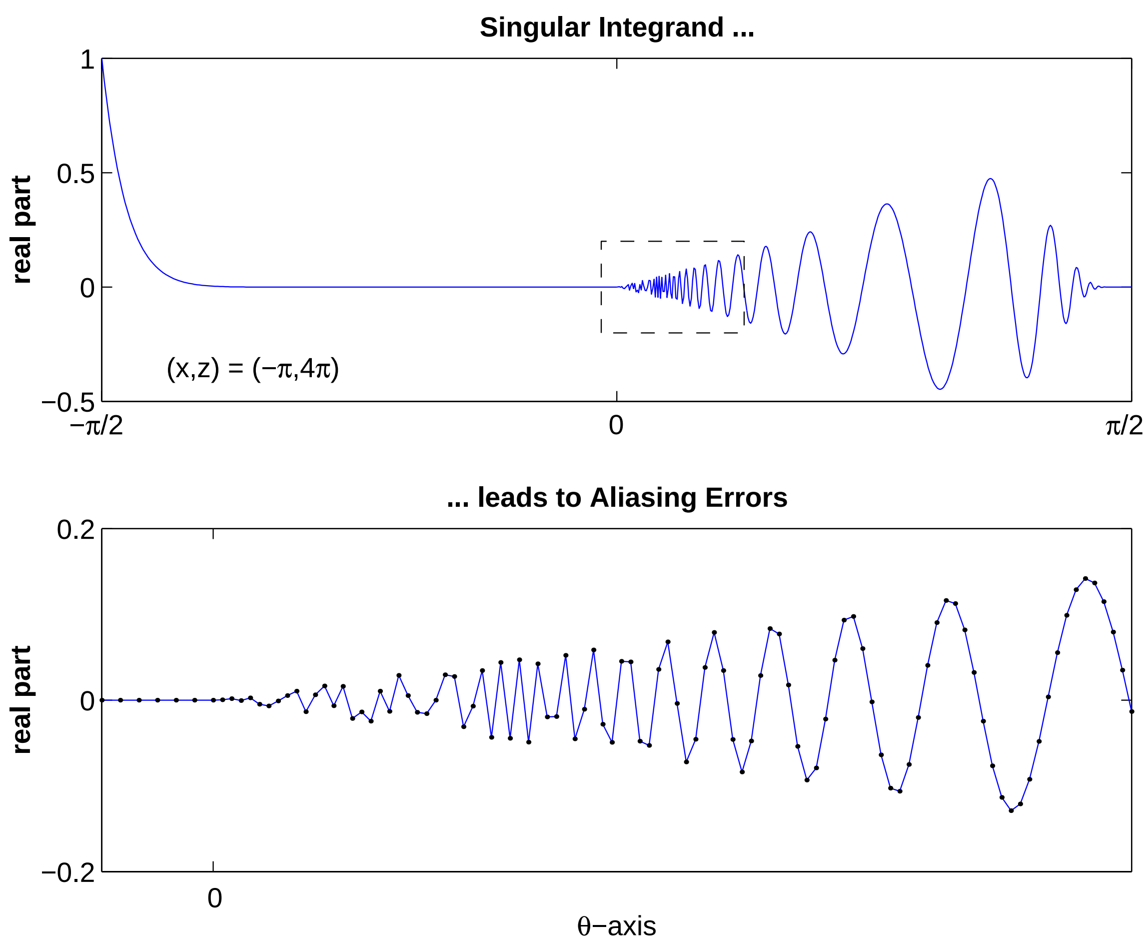
Desingularization I

Singular Exponent

- vertical wavenumber $m(k) \longrightarrow \infty$, as $k \longrightarrow k_f^+$
- Queney's trigonometric coordinates

$$k = \begin{cases} -k_f \sin\theta & \text{for} \quad -\frac{\pi}{2} \le \theta \le 0 \quad \text{(decay)} \\ k_f \sec\theta & \text{for} \quad 0 < \theta < \frac{\pi}{2} \quad \text{(waves)} \end{cases}$$

- amplitude of integrand $\longrightarrow 0$, as $\theta \longrightarrow 0^+$



Desingularization II

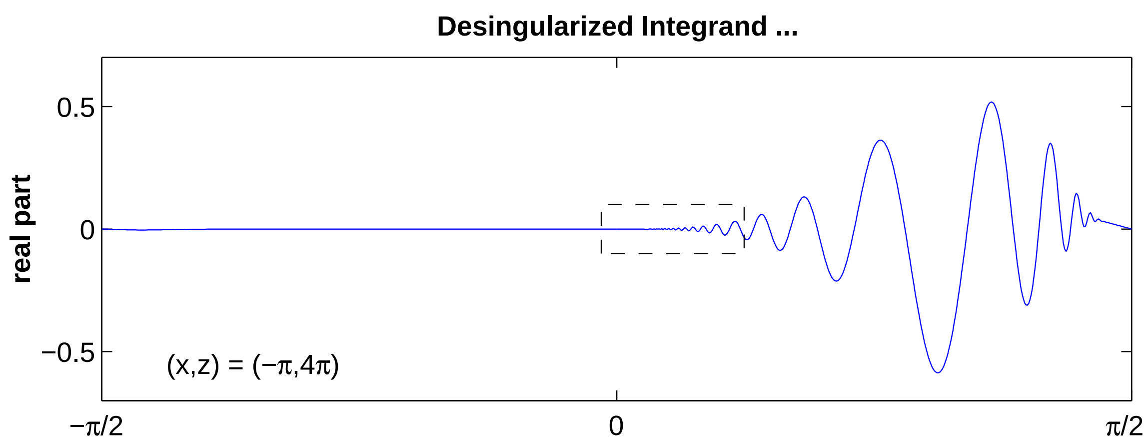
Numerical Errors

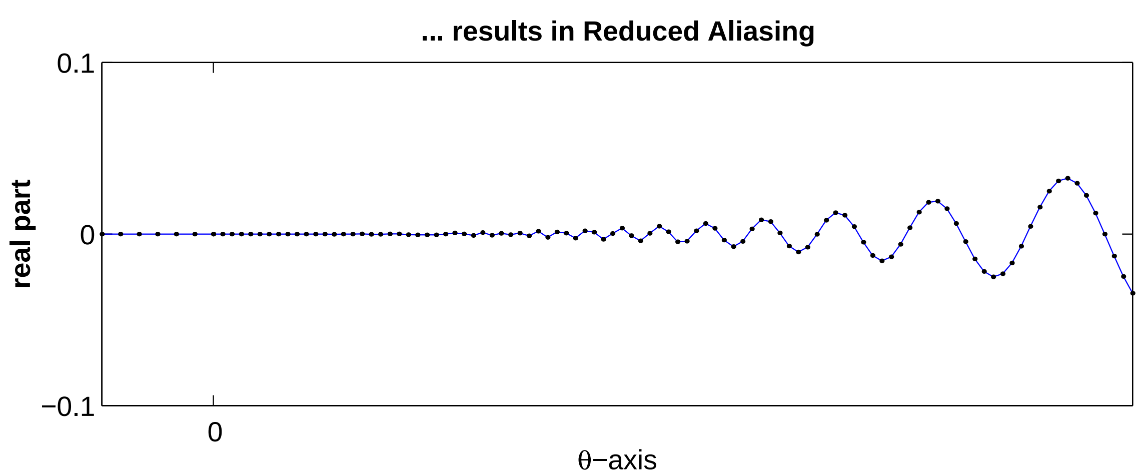
- FFT-based quadratures have periodicity problems

  $\longrightarrow$ wrap-around from slow decay of downstream wake
- aliasing errors

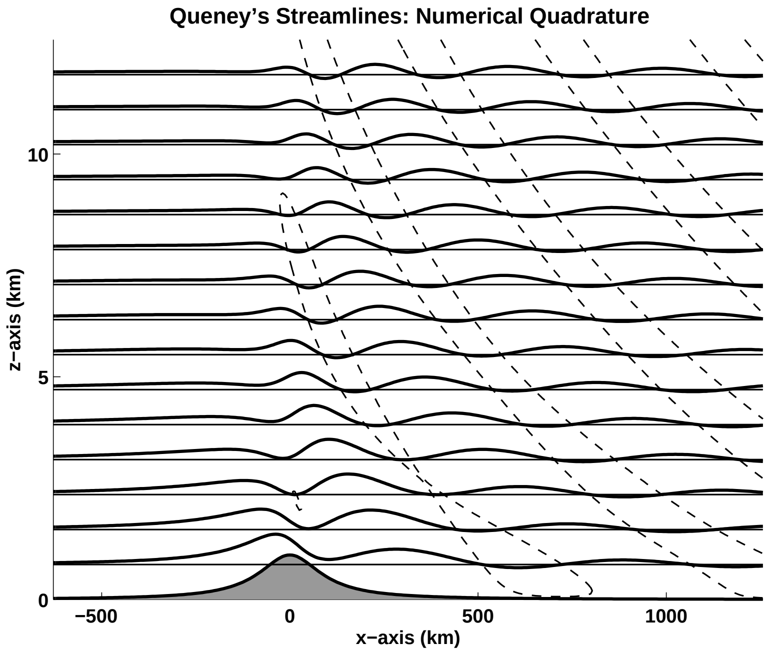
  $\longrightarrow$ upstream wavy artifacts & downstream interference
- evaluate $\mathcal{E}_n$-integrals using *exponential integral*, $Ei(x)$

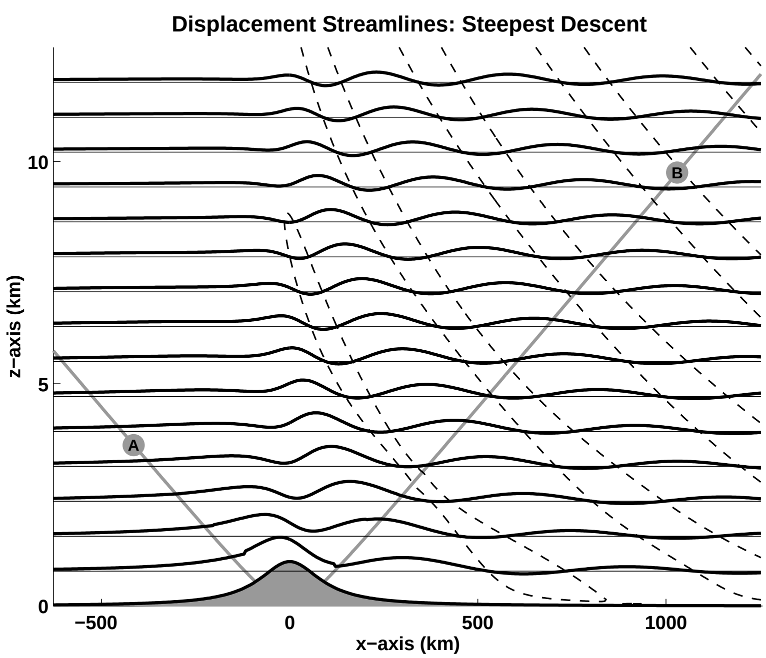
$$\mathcal{E}_n = \int_0^{\pi/2} e^{ik_S z \csc\theta} \ \sin^n\theta \ \cos\theta \ d\theta$$





Steepest Descent Approximation



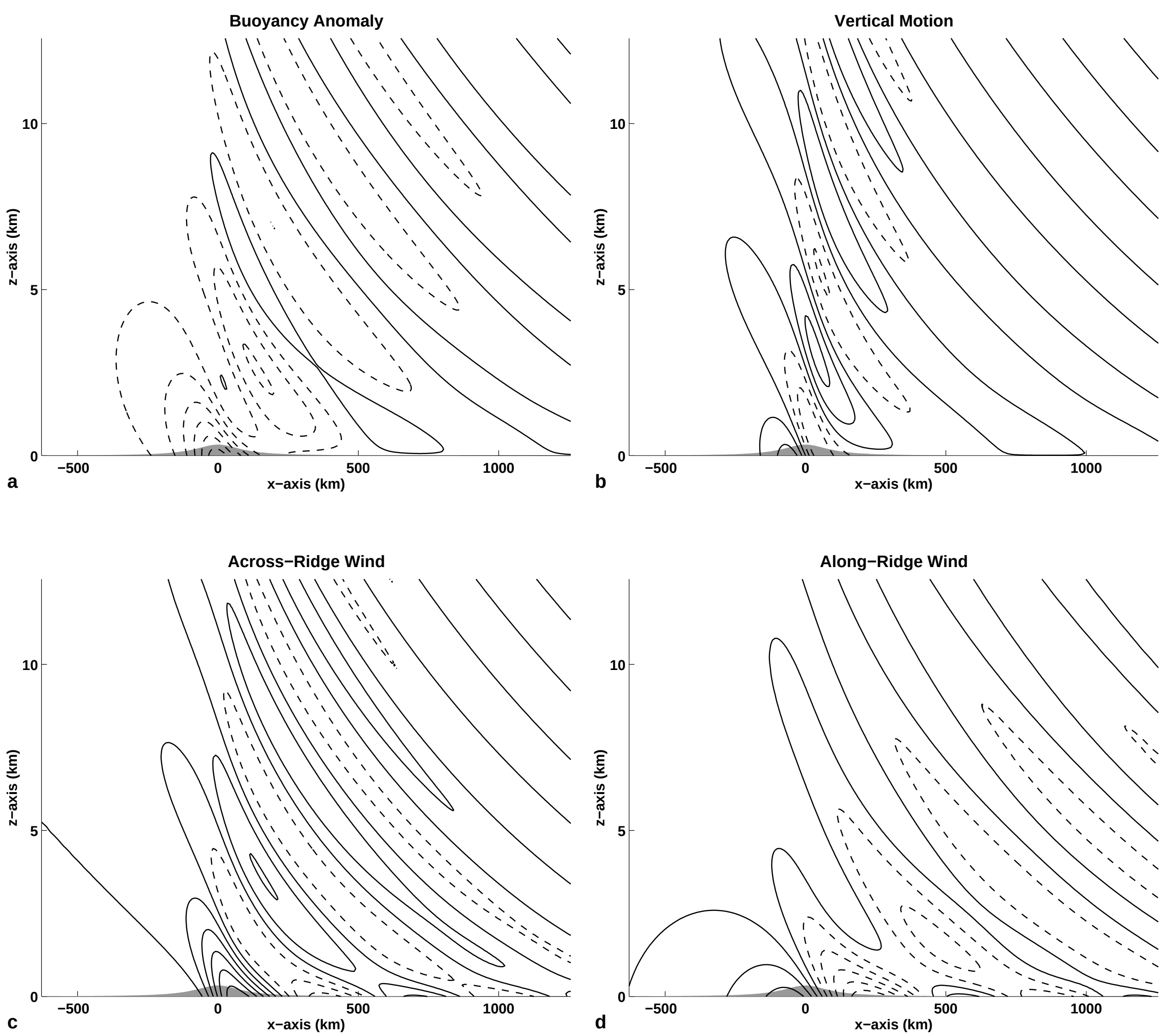


- decay of wave amplitude in zenith

$$\text{waves} \propto (\mathcal{R}z)^{1/6} \exp\left\{-C\,\mathcal{R}^{-2/3}z^{1/3}\right\}$$

Other Fields

- desingularized quadratures for velocity & vertical motion
- $\mathcal{R} = 1.0$, $\mathcal{F} = 3.0$



3D Topography

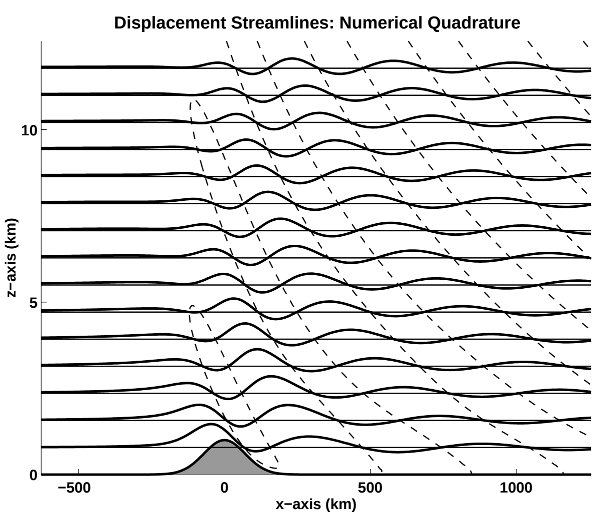
Flow Past a Circular Gaussian Mountain

- 3D linear dispersion relation

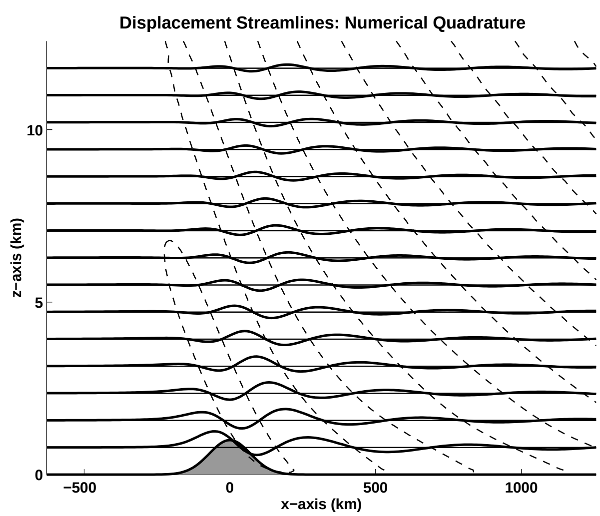
$$m(k,l) = \begin{cases} ik_s\sqrt{\dfrac{k^2+l^2}{k_f^2-k^2}} & \text{for} \quad 0 \le k < k_f \\ k_s\sqrt{\dfrac{k^2+l^2}{k^2-k_f^2}} & \text{for} \quad k_f < k < \infty \end{cases}$$

- same desingularization integrals apply
- displacement streamlines: $\mathcal{R} = 1$, $\mathcal{F} = 1$

2D gaussian ridge



3D gaussian mountain



Circular Mountain

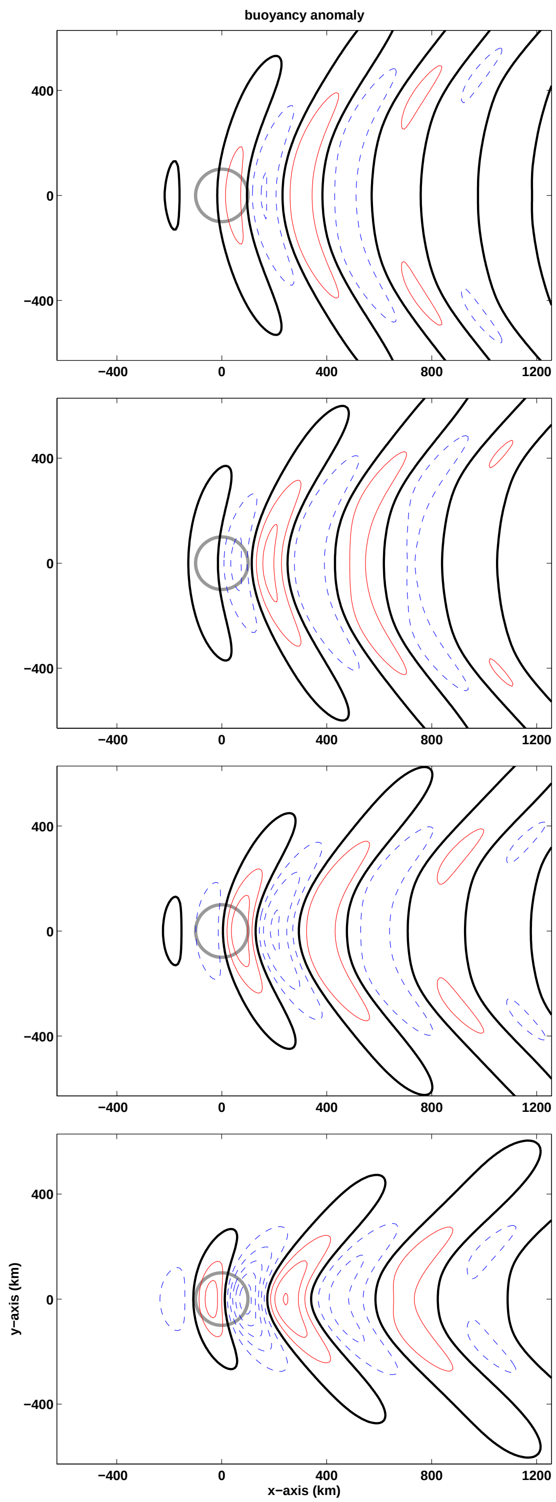
- buoyancy anomaly: $\mathcal{R} = 1$, $\mathcal{F} = 1$

$z = 2.0\,\pi$

$z = 1.5\,\pi$

$z = 1.0\,\pi$

$z = 0.5\,\pi$

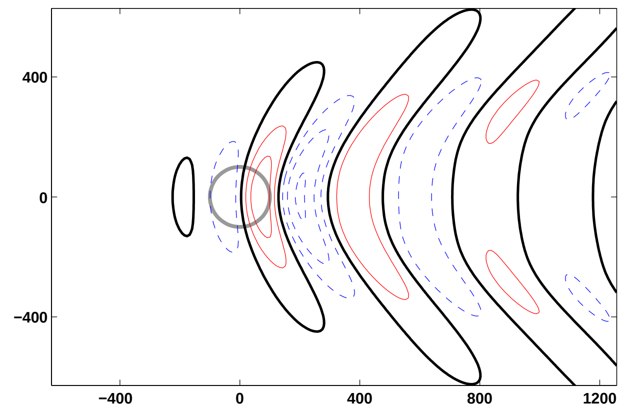


Transition to QG

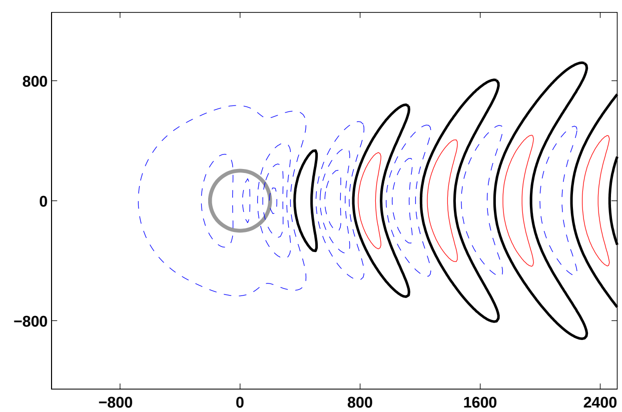
- buoyancy anomaly at $z = \pi$: $\mathcal{F} = 1$
- $\mathcal{R} \longrightarrow 0$: by $\nearrow$ mountain scale

  $\longrightarrow$ development of QG anticyclone

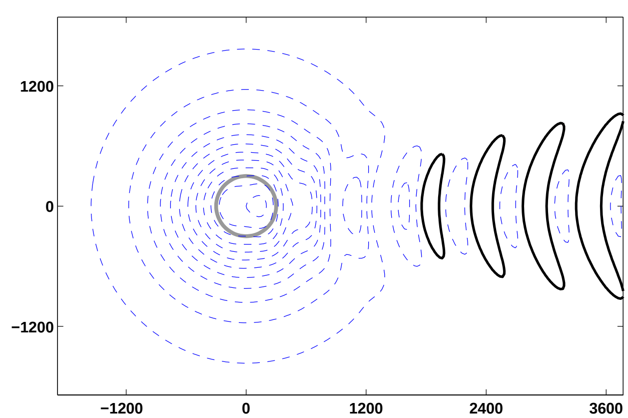
  $\longrightarrow$ wave amplitude $\searrow$ like $e^{-1/\mathcal{R}}$?? (contour int $\searrow$)



$\mathcal{R} = 1$



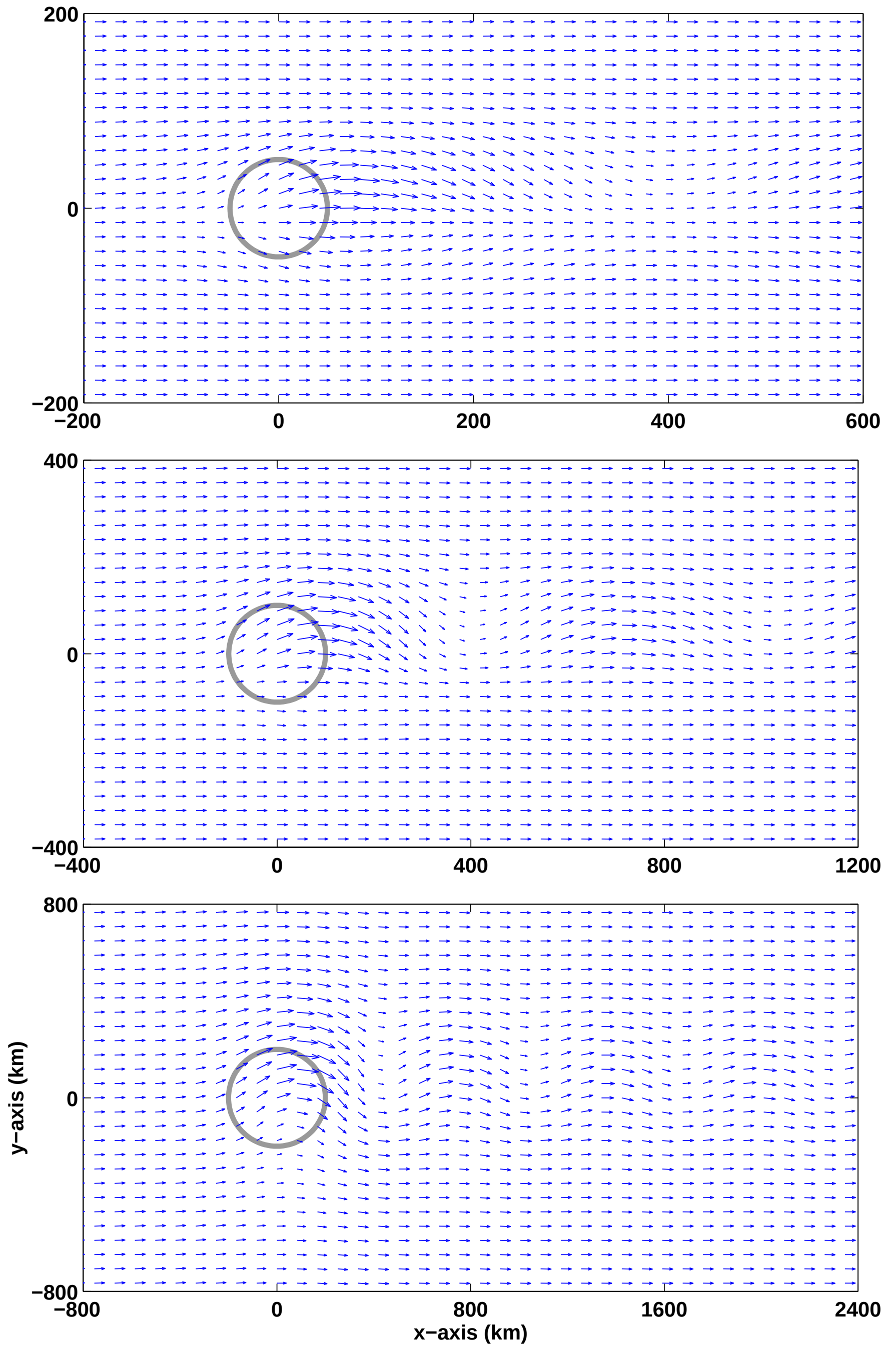
$\mathcal{R} = 1/2$



$\mathcal{R} = 1/3$

Transition to QG

- surface wind vectors: $\mathcal{F} = 1$
- transition from split flow to anticyclone as $\mathcal{R} \longrightarrow 0$



$\mathcal{R} = 2.0$

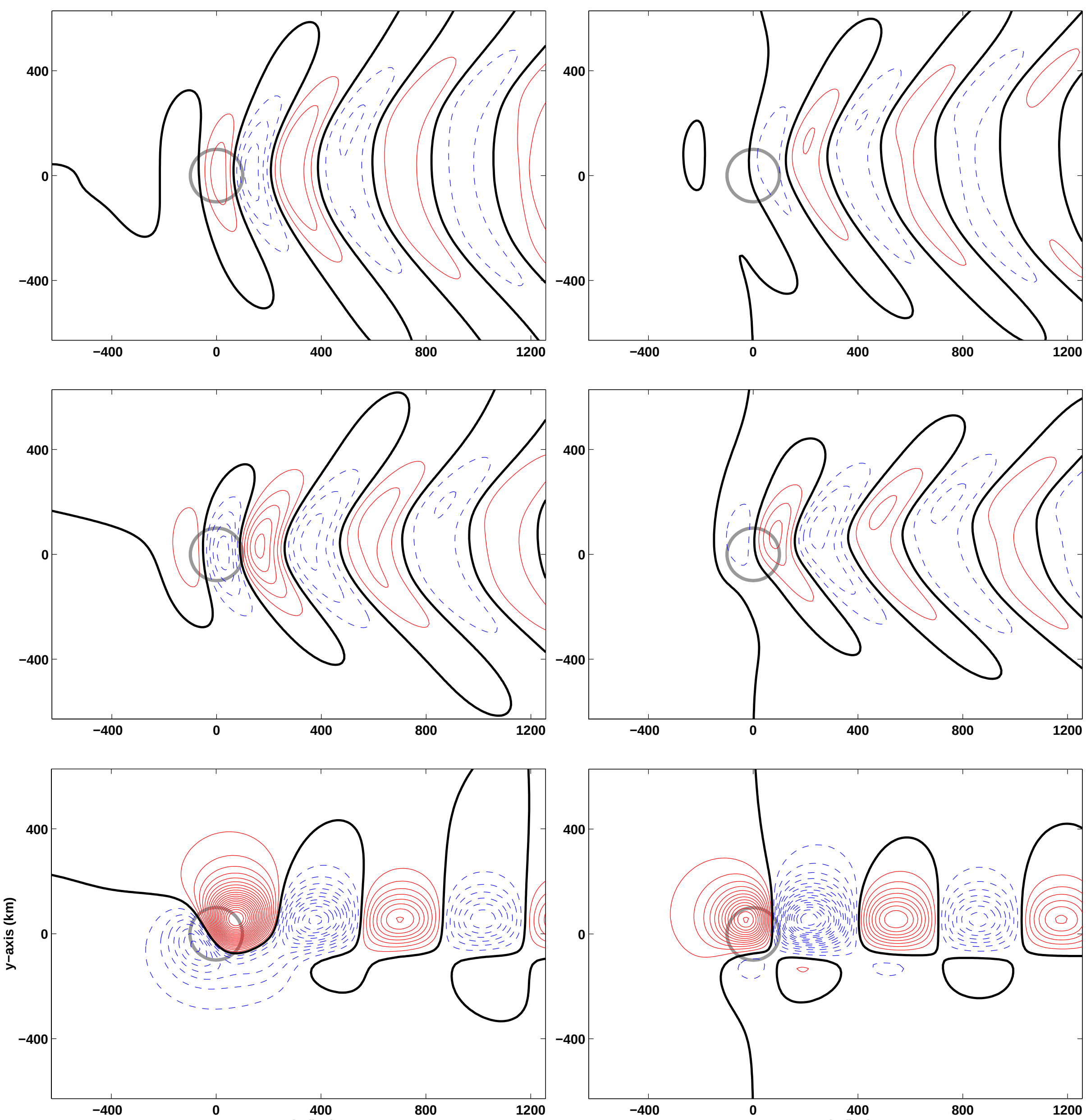
$\mathcal{R} = 1.0$

$\mathcal{R} = 0.5$

Disturbance Winds

- $\mathcal{R} = 1$, $\mathcal{F} = 1$ at heights $z = \pi, \dfrac{\pi}{2}, 0$ km

u-winds $\qquad$ v-winds



Finale

Topographic Flow with Rotation

- flow structures consistent with non-rotating & QG
- desingularized Fourier quadratures: 2D & 3D