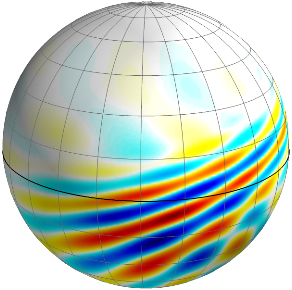
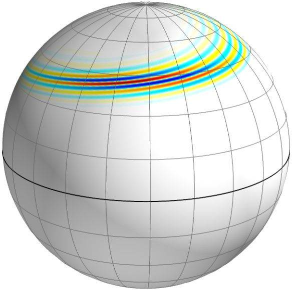
Rossby Wave Modes for Rotating Shallow Water on the Sphere

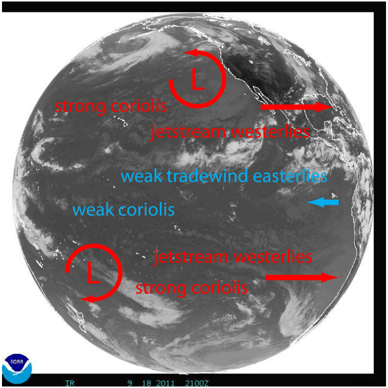
- midlatitude waves & critical latitude exclusion from the tropics
- equatorial β-plane & tropical waveguide
- finite number of global Rossby waves observed in climatology

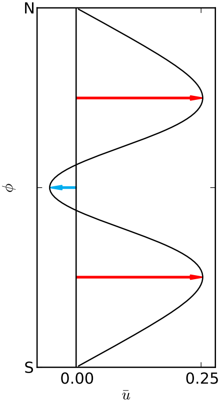




- David J Muraki & Kevin Mitchell, Simon Fraser University

Midlatitude & Equatorial Regions





Zonal Winds & Coriolis Rotation

- midlatitudes: strong jetstream westerlies & strong coriolis effect
- equatorial (dry) tropics: weak tradewind easterlies & weak coriolis effect (sign change)

Three Communities of Rossby Wave Theory

Midlatitudes (meridionally-sheared jet)

- ▷ midlatitude β-plane, slow waves at wavelengths scaling on Rossby radius
- ▷ diminishing of midlatitude jet $\rightarrow$ NO propagation into tropics
  - ▹ critical latitude (zonal phase speed $=$ angular wind speed), Dickinson (1970)

Tropics (negligible shear)

- ▷ equatorial β-plane theory, equatorially-trapped modes
- ▷ infinite number of discrete modes, at all wavelength scales, Matsuno (1966)

Global

- ▷ long-wavelength, planetary-scale waves
- ▷ finite number observed in climatolological data, Madden (2007)
  - ▹ projection onto spherical harmonics
  - ▹ link with limited number of computed discrete modes, Kasahara (1980) ?

How are these perspectives reconciled into one framework?

Rotating Shallow Water (rSW) on the Sphere

Laplace Tidal Equations

- winds, $\vec{u}$ & height field, $H = 1 + \epsilon\, \eta$
- spherical longitude-latitude coordinates, (λ, ϕ)

$$\epsilon^2 \frac{D\vec{u}}{Dt} + (\sin\phi\; \hat{r}) \times \vec{u} = -\,\epsilon\, \nabla\eta$$

$$\epsilon \frac{D\eta}{Dt} + (1 + \epsilon\,\eta)\; (\nabla \cdot \vec{u}) = 0$$

- atmospheric scaling: small Rossby number asymptotics

$$\epsilon = \left(\frac{4\Omega^2 r^2}{gH_0}\right)^{-1/2} \ll 1$$

A Long History of Linear Waves . . .

- . . . Margules (1892/93), Hough (1897/98) . . . Longuet-Higgins (1968) . . .
- wave modes: inertia gravity (fast), mixed-Rossby/gravity, Kelvin & Rossby (slow)
- midlatitude waves: Rossby (1939/40) . . .
- equatorially-trapped waves: Matsuno (1966) . . .

Linear Wave Modes

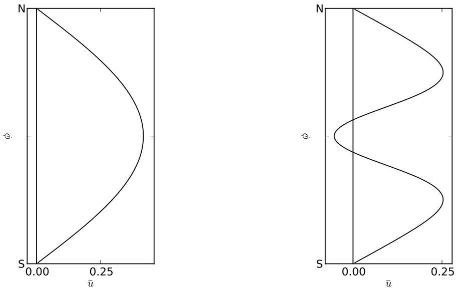
Background Zonal Wind

- linearize rSW about midlatitude jet & tradewind profile, $\bar{u}(\phi)$

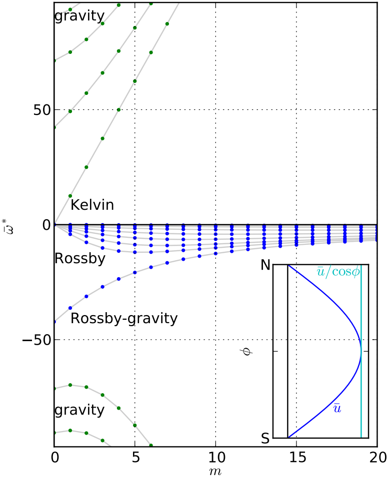
$$\begin{pmatrix} u \\ v \\ h \end{pmatrix} = \begin{pmatrix} \bar{u}(\phi) \\ 0 \\ \bar{h}(\phi) \end{pmatrix} + \begin{pmatrix} \hat{u}(\phi) \\ \hat{v}(\phi) \\ \hat{h}(\phi) \end{pmatrix} e^{i(m\lambda - \omega t)}$$

- non-constant coefficient eigenvalue problem for $\omega_n(m)$
  - FFT spectral computing & WKB analysis
- dispersion relation & wave spectrum comparison:

super-rotation (no shear) tradewind (climatological shear)



Super-Rotation Profile → Standard Spectrum



▷ reduced frequency:

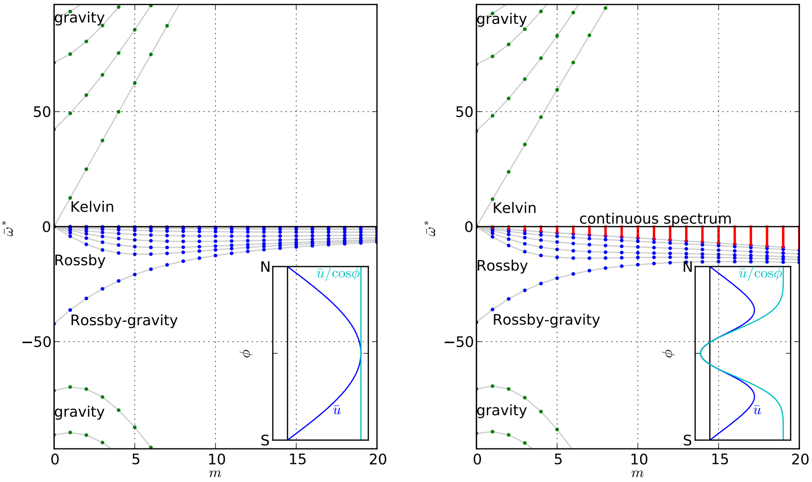
$$\bar{\omega}^* = \omega - m \left(\frac{\bar{u}(\phi)}{\cos \phi} \right)_{\text{max}}$$

▷ slow Rossby & fast gravity waves

▷ $\epsilon = 0.084 \ll 1$

▷ discrete eigenvalue spectrum, $n = 0 \ldots \infty$

Super-Rotation vs Tradewind Profiles



- tradewind profile ⇒ continuous spectrum, Case (1960) & Farrell (1982)

Tradewind Profile → Singular Spectrum

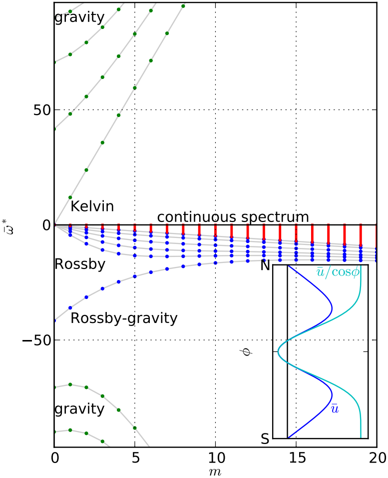
- reduced frequency:

$$\bar{\omega}^* = \omega - m \left(\frac{\bar{u}(\phi)}{\cos\phi} \right)_{\text{max}}$$

  - $\epsilon = 0.084 \ll 1$

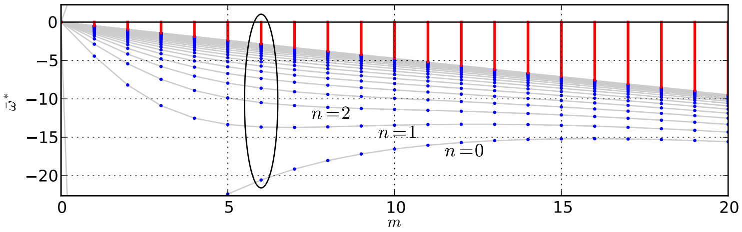
- discrete spectrum, $n = 0 \ldots \infty$
- continuous spectrum:

$$\left(\frac{\bar{u}(\phi)}{\cos\phi} \right)_{\text{min}} < \frac{\omega}{m} < \left(\frac{\bar{u}(\phi)}{\cos\phi} \right)_{\text{max}}$$



- tradewind shear introduces two *infinite* sets of Rossby wavemodes

Three Flavors of Rossby Wave Modes?



Discrete Rossby Spectrum

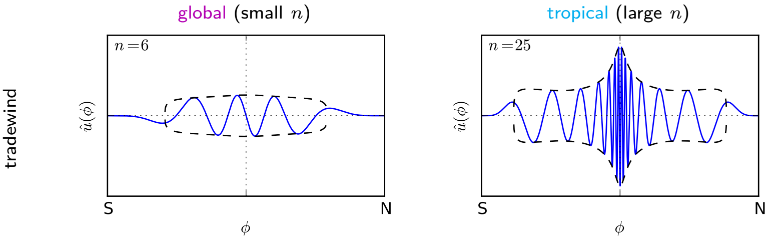
- global planetary-scale waves, accounted for in Madden (2007)?
- tropical shortwaves . . . equatorial-β, but longwave in midlatitudes

Continuous Rossby Spectrum

- midlatitude shortwaves, all have critical latitudes
- zonal phase speed lies in (angular) wind band:

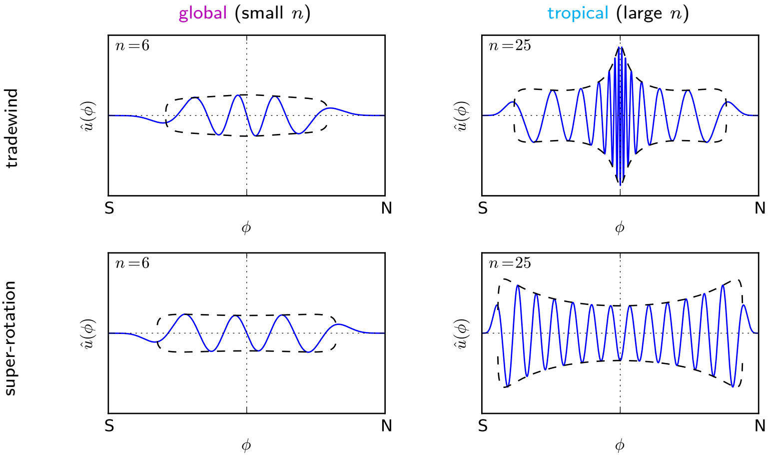
$$\left(\frac{\bar{u}(\phi)}{\cos\phi}\right)_{\text{min}} < \frac{\omega}{m} < \left(\frac{\bar{u}(\phi)}{\cos\phi}\right)_{\text{max}}$$

Discrete Eigenmode, Meridional Rossby Wave $(m = 6)$



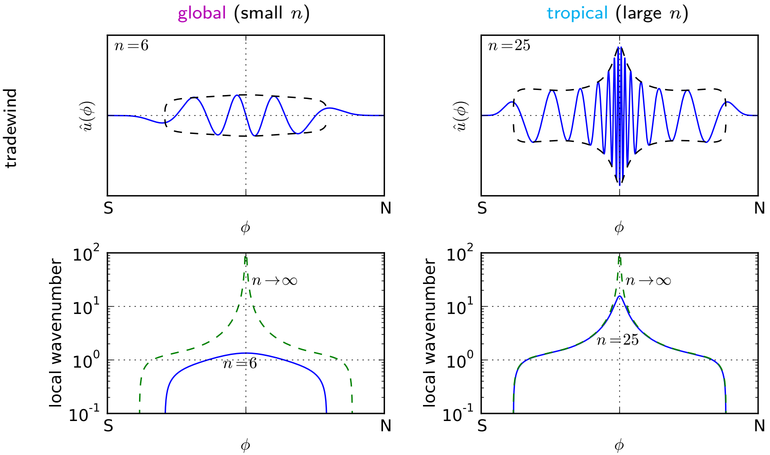
- ▷ global planetary-scale wave, relatively uniform wavelength
- ▷ tropical shortwave, extremely latitude-dependent wavelength

Discrete Eigenmode, Meridional Rossby Wave $(m = 6)$



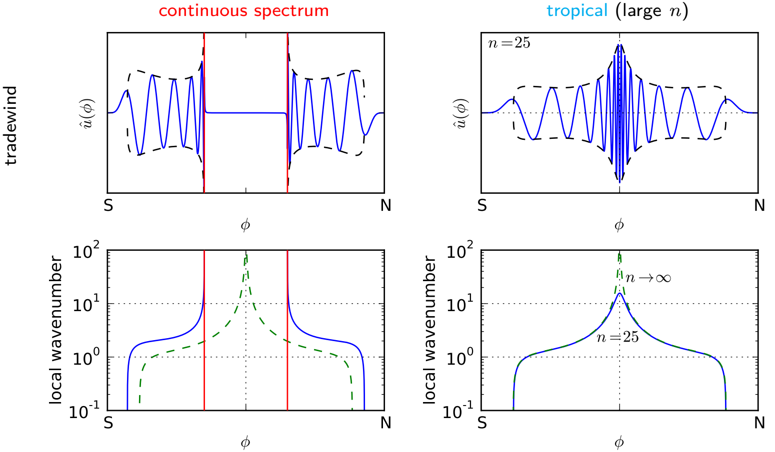
- global planetary-scale wave, similar to spherical harmonic, Hough mode, etc
- tropical shortwave . . . but longwave in midlatitudes (not in equatorial-β theory)

Discrete Eigenmode, WKB Analysis $(m = 6)$



- WKB analysis of tropical shortwave, $n \to \infty$:
  - maximum limit of wavenumber in midlatitudes, unbounded at equator

Continuous Singular Mode, WKB Analysis $(m = 6)$



- ▷ midlatitude shortwave: wavescales shorter than discrete modes
- ▷ exclusion of waves from tropics by critical latitude (meridional group velocity $\to$ 0)

Oscillatory Asymptotics

- introduce fast meridional phase $\rho(\phi)$ with $\epsilon\rho'(\phi) = O(1)$:

$$\begin{pmatrix} u \\ v \\ h \end{pmatrix} = \begin{pmatrix} \bar{u}(\phi) \\ 0 \\ \bar{h}(\phi) \end{pmatrix} + \vec{A}(\phi)\ e^{i\rho(\phi)}\ e^{i(m\lambda - \omega t)}$$

- local dispersion relation $\bar{\omega}(\phi) = \bar{\omega}(m, \rho')$ with generalized wavenumber $\rho'(\phi)$:

$$\frac{\bar{\omega}}{m}\left(\frac{(\epsilon m)^2}{\cos^2\phi} + (\epsilon\rho')^2 + \frac{\sin^2\phi}{\bar{h}}\right) + \frac{\bar{h}}{\cos\phi}\left(\frac{\sin\phi}{\bar{h}}\right)_\phi = 0$$

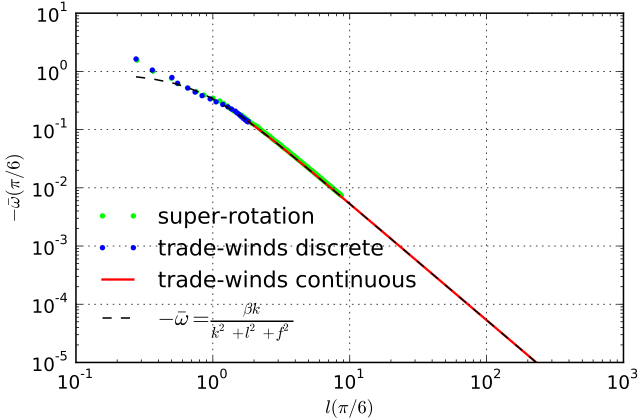
- contains key features of spectrum:
  - turning points: equatorial-trapping & infinite number of discrete eigenmodes
  - critical latitudes: singular point & continuous modes
- midlatitude scalings . . .

$$\bar{\omega} \to \epsilon\bar{\omega} \quad ; \quad \frac{\epsilon m}{\cos\phi} \to k \quad ; \quad \epsilon\rho' \to l \quad ; \quad \frac{\sin^2\phi}{\bar{h}} \to f^2$$

- . . . recover midlatitude Rossby wave dynamics, Hoskins/Karoly (1981)

$$\omega \approx Uk - \frac{\beta k}{k^2 + l^2 + f^2}$$

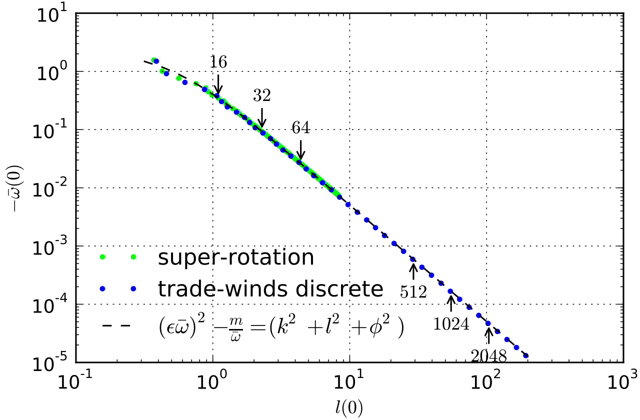
Local Dynamical Equivalence $(m = 6)$



Midlatitude β-Plane Theory $(\phi = \pi/6)$

- ▷ equivalent dispersion relation: $\bar{\omega}_n(m)$ as function of local wavenumber
- ▷ super-rotation & tropical shortwave + midlatitude shortwave dynamics coincide

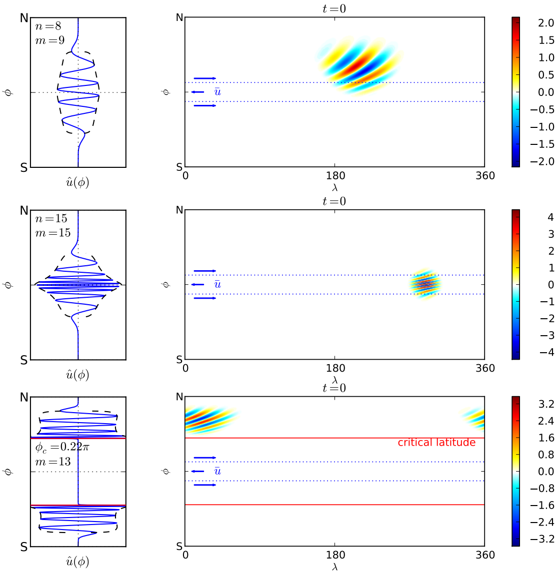
Local Dynamical Equivalence $(m = 6)$



Equatorial β-Plane Theory $(\phi = 0, \epsilon = 0.084 \ll 1)$

- equivalent dispersion relation: $\bar{\omega}_n(m)$ as function of local wavenumber
- super-rotation & tropical shortwave dynamics coincide

Rossby Wavepacket Propagation



global

planetary scale wave

crosses tropics

tropical

tropical shortwave

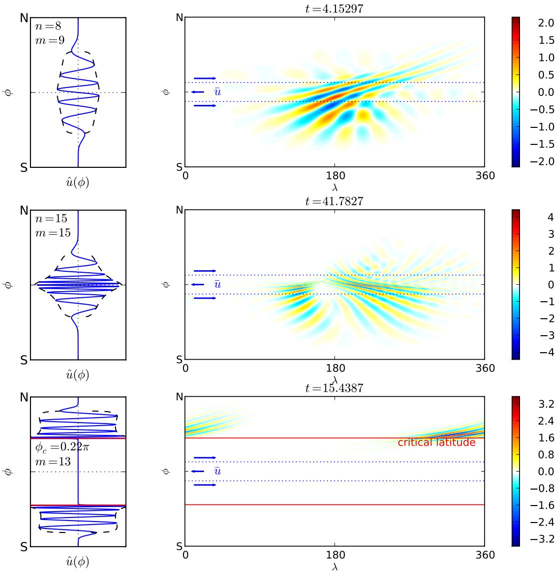
midlatitude longwave

midlatitude

critical latitude

exclusion from tropics

Rossby Wavepacket Propagation



global

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midlatitude

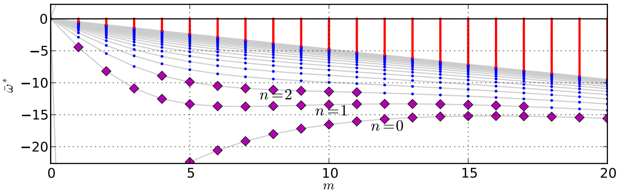
critical latitude

exclusion from tropics

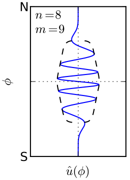
Rossby Wave Modes for rSW on the Sphere

Midlatitude, Tropical & Global Waves

- dynamics consistent with local β-plane, $\omega(k, l)$
- discrete spectrum accounts for equatorial-crossing waves (+ numerical quirks)
- continuous spectrum accounts for critical latitude behavior



Rossby Wavepacket Propagation (movies)



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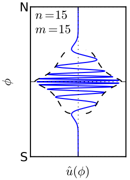
global

planetary scale wave

crosses tropics



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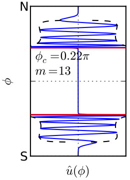
tropical

tropical shortwave

midlatitude longwave



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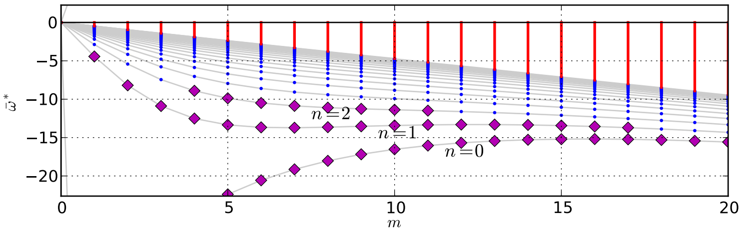
midlatitude

critical latitude

exclusion from tropics

Global Modes



Which Global Modes are Identifiable by Spherical Harmonics?

- global planetary-scale waves, accounted for in Madden (2007)?
- which discrete modes have a unique index correspondence with a spherical harmonic?
  - only longest meridional wavelengths due to extreme shortwave behavior
  - fewer matched modes with increasing ϵ