

Going Low: part 3

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In the first two parts of this series, I discussed 7-card stud high-low with an 8-low qualifier. I am going to switch to Omaha high-low for the rest of the series. This game also has an 8-low qualifier. One basic question that players sometimes ask is how frequently boards occur that allow a low. Let's take a look at this question and go through the argument that produces an answer.

I assume that none of the players' hands are known. In other words, I am working this out from the standpoint of the so-called neutral observer. The total number of possible boards is $C(52, 5) = 2,598,960$. In order for a board to allow a low, there must be at least three distinct ranks chosen from $\{A,2,3,4,5,6,7,8\}$. Henceforth, we shall call any card of one of those ranks a low card, and we shall call all other cards high cards.

Let's list the types of boards that allow a low:

- three cards of distinct low ranks and two high cards,
- four low cards, including a pair, and one high card,
- four cards of distinct low ranks and one high card,
- five low cards including 3-of-a-kind,
- five low cards including two pairs,
- five low cards including one pair, and
- five cards of distinct low ranks.

For the first type above, the three low cards are of distinct ranks. There are $C(8, 3) = 56$ choices for the low ranks, there are 4 ways of choosing each of the low cards of a given rank, and there are $C(20, 2) = 190$ ways of choosing two high cards from 20. Multiplying all these numbers gives 680,960 boards of this type.

Next we consider boards of the second type above. There are 8 choices for the rank of the pair, there are 6 choices of pairs of the given rank, there are $C(7, 2) = 21$ ways to choose the remaining 7 low ranks, there are 4 cards for each of the other 2 low ranks, and there are 20 choices for the high card. Multiplying yields 322,560 boards of this type.

Boards of the third type above have four low cards of distinct ranks. There are $C(8, 4) = 70$ choices for the 4 ranks, there are 4 choices of card for each chosen low rank, and there are 20 choices for the high card. Multiplying the numbers produces 358,400 boards of this type.

The fourth type of board above has 8 choices for the rank of the trips, 4 choices for the trips, the other 2 ranks are chosen from 7 ranks in $C(7, 2) = 21$

ways, and there are 4 choices for each card of the other two ranks. Upon multiplying the numbers, we have 10,752 boards of this type.

The fifth type of board above has $C(8, 2) = 28$ choices for the ranks of the two pairs, with 6 ways of choosing each pair, and 24 choices for the remaining card. This yields 24,192 boards of this type.

The sixth type of board has a single pair among the 5 low cards. There are 8 choices for the rank that is paired and 6 choices for the pair. There are $C(7, 3) = 35$ choices for ranks of the other three cards, and 4 choices for each of the three ranks. Multiplication gives 107,520 boards of this type.

Finally, the sixth type has five low cards of distinct ranks. There are 5 choices for the ranks from 8 possible ranks, and for each rank there are 4 choices for cards. This produces 57,344 boards. Adding the numbers of boards of the types allowing a qualifying low yields 1,561,728. Dividing this number by 2,598,960, the total number of boards, gives us 0.601 as the probability of a low board occurring for Omaha high-low with an 8-low qualifier. Thus, a low is possible 60% of the time.

The steps we have just gone through afford an excellent example of how one solves a typical counting problem arising in poker. The first step is to identify exactly what must be counted. The second step is to break the problem into smaller subproblems. The third step is to solve each of the subproblems by making a sequence of choices and multiplying the numbers coming out of the choices. The last step is to check your work, but that step I have not shown here. I have done this last step in the appropriate file at my website.