So Many Ties!

Brian Alspach

Brett James of Baltimore wrote to me with the following story. "Recently, a friend of mine was involved in a hand of Texas Hold'Em, where three of the eight players were dealt J-9 and the flop came J-9-x. I was wondering if you might be able to tell us the chances of that happening. I have a modest background in statistics and wasn't able to come up with an answer. I can compute the chances of any given person(s) getting a certain hand, but did not know how to incorporate the flop into the calculations. If you have the time, any help would be appreciated."

I wish to thank Brett for sending the question to me as it motivates this article. I also suspect other readers will find it of interest because similar situations do occur now and then.

The calculation is straightforward if you have access to a program that can handle big numbers easily. The idea is to count the total number of eight-player partial semideals and then count the number of partial semideals that satisfy the property in which we are interested. In this case the property is three players being dealt J-9 and a J-9 coming on the flop. Let me remind you of the difference between a partial semideal and a partial deal. A partial semideal implies that we are interested in what the eight hands and the flop are, but we do not care which players get which hands. If the original question had asked for particular players being dealt certain hands, then we would work with partial deals instead of partial semideals.

The first step is to count the total number of partial semideals for eight players. We first choose 16 cards to be dealt to the players. This can be done in $C(52,16) = 10,363,194,502,115$ ways. Once the 16 cards are chosen to be dealt to eight players, we must break them up into eight hands of two cards each. This can be done in $15!! = 2,027,025$ ways. The last choice is the three cards comprising the flop. We are choosing three cards from 36 so that there are $C(36,3) = 7,140$ possible flops. Multiplying these three numbers gives us the total number of partial semideals. Performing the multiplication yields 149,986,083,956,538,557,227,500 partial semideals

The next step is to count the partial semideals satisfying the J-9 criteria. There are four ways we can choose three jacks to be dealt to players. Similarly, there are four ways we can choose three nines to be dealt to the players. Once the three jacks and three nines are chosen, there are three choices for a nine to go with a particular jack, leaving two choices for a nine to go with another particular jack, and no choice for the last nine to go with a jack. This means there are 96 ways we can form three player hands of the form J-9. We still need five more player hands. For the other five hands, we are choosing ten cards from 44 (we can't choose the remaining 9 or J). This gives us $C(44, 10) = 2,481,256,778$ choices. The ten cards can be split into five hands in $9!! = 945$ ways. Because the flop must contain the remaining J and 9, there is no choice for them. The third card in the flop can be chosen from 34 cards giving us 34 possible flops.

We multiply the preceding numbers together and get 7,653,386,906,605,440 partial semideals with the J-9 property about which Brett asked. We divide this number by the total number of partial semideals above and obtain a probability of about 1 in 19,597,348 that it would happen. If we go through the same calculations for ten players, the chances of this happening are about 1 in 9,145,429.

There are several observations we can make about this phenomenon. There are 78 distinct pairs of different ranks. Because the flop can produce three people flopping the same two pairs for only one pair of distinct ranks, the chances that the flop gives three players the same two pairs (in a 10-handed game) is about 1 in 117,249. Of course, the latter number is not a good reflection of reality because it is highly unlikely that three people holding 2-7 are all going to see a flop of 2-7-x. What is more likely is that once the hand is over (unless there are several players with poor poker etiquette) the players will start mentioning that they would have flopped two pairs. Eventually the story will emerge that three people would have flopped two pairs and someone will say, "I wonder what the chances are of that happening?"

On the other hand, three players holding hands like A-K, A-Q, etc. are likely to see the flop and now and then there will be three players with the same good starting hands flopping the same two pairs. Everyone will have a good laugh about it and someone will say, "I wonder what the chances are of that happening?"

I don't recall ever seeing the above situation over the thousands of hands I have played, but I do remember the following situation. I was in the small blind in the last hand before the first break at one of the major tournaments at Casino Regina. There was a raise and two callers when the action returned to me. I peeked at my cards and found A-10. I decided to call the raise. The flop came 10-10-10 and as I was first to act, this was a good situation to slow play the monster flop. At the end of the hand, I learned that all three of my opponents held A-K so that they were happy to check the flop and the turn. Each of them was, of course, looking for an A or a K. I finally bet after the river card was another blank. One of the players actually called me so that I did win some chips.