

Is There a Flush Out There: Part II?

Brian Alspach

In Part I we introduced the method of inclusion-exclusion for determining exact probabilities that at least one random hand has a flush under various conditions on the board for both hold'em and Omaha. We gave an example, but for the uninitiated, it is unlikely a single example is going to lead to deep understanding of the method. In this article, we concentrate on the actual results and display them in a sequence of tables. All these results are obtained using inclusion-exclusion.

Given boards with three, four or five suited cards, the entries in the first table give the probabilities that one or more hands, from ten randomly dealt hold'em or Omaha hands, have a flush. This is what a neutral observer would see.

Suited cards on board	Hold'Em	Omaha
3	.3656	.964
4	.9966	.923
5	1	.8542

NEUTRAL OBSERVER

The above values are of interest because they give an overall shape to what we can expect when the board allows flushes. Taking a quick look at the table, we note that flushes are a potential threat in most situations. The entry of a 1 for hold'em with five suited cards on board reflects the fact that all players then have a flush.

Hold'em with three suited cards on board is the only situation where the probability of at least one random hand containing a flush is not huge. It is slightly more than once in three. All the other situations frequently produce random hands with a flush. Even in Omaha with five suited cards on board, almost seven out of eight times there will be a hand making a flush. These numbers partially explain why sets lose to flushes so often in Omaha when the board allows a flush and does not pair.

Of more interest are the conditional probabilities for a given player. The next table is for hold'em. The first column tells us the number of suited cards on board. Since the game is hold'em, we omit the case of five suited cards on board because all players have a flush. (The pertinent question then becomes whether someone else has a better flush, and that is the topic of our article in next month's issue.) The other columns list how many cards from the flush suit a fixed player X has in her hand. The entries in the table give the probability that at least one of the other nine random hands has a flush.

Suited on Board	Suited for Player		
	0	1	2
3	.3605	.2977	.2379
4	.9947	.9897	.9804

HOLD'EM FROM PLAYER'S VIEWPOINT

Omaha is more complicated because a player may have anywhere from zero to four cards in the suit for which the board allows a flush. The entries in the next table give the probability that at least one of the nine random opponent hands has a flush given that the board and the player have the corresponding numbers of cards in the suit for which a flush is allowed.

Suited on Board	Suited for Player				
	0	1	2	3	4
3	.974	.9396	.8779	.7816	.65
4	.9396	.8779	.7816	.65	.4918
5	.8779	.7816	.65	.4918	.3252

OMAHA FROM PLAYER'S VIEWPOINT

Random hands have been used here because exact values are then easy to calculate. The problem is that random hands introduce a distortion into any given situation. It is natural to ask whether there is much value in looking at these numbers. The question is reasonable because when you actually are playing poker, you are not playing against random hands. Your opponents have made decisions about folding or remaining in the action based on many considerations.

I believe they have some value, but player's must exercise care in how they use them. For one thing, the numbers give a rough indication of what is and what is not worth worrying about. For example, an Omaha game in which many players are seeing the flop and many players like hands with flush possibilities should make a player wary when the board allows a flush and the player cannot beat a flush.

I also think there is value in comparing the numbers for hold'em and Omaha for various situations. For example, a player who has developed a good intuitive feel about playing hold'em hands when the board allows flushes can see how this stacks up against the probabilities for random hands. She can then extrapolate from the random hand probabilities for Omaha to gain some insight for playing Omaha when the board allows flushes.