Flushing Boards and Random Hands

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Abstract

Given certain boards in hold'em and Omaha, we examine the probabilities of randomly dealt hands holding flushes.

Hold'Em

Suppose we have a board in hold'em with three suited cards and two cards not in the suit. If we now deal 10 random hands, what is the probability at least one player has been dealt two cards that make a flush with the board? For ease of discussion, we'll say that the board has exactly three hearts.

We are going to use the principle of inclusion-exclusion for probabilities. Label the players $1, 2, \ldots, 10$. Let $P(i)$ be the probability that player i has been dealt two hearts. It is easy to see that

$$P(i) = \frac{10 \cdot 9}{47 \cdot 46} = \frac{45}{1,081}.$$

Now let $P(ij)$ be the probability that players i and j both are dealt two hearts. We just saw the proability that player i is dealt two hearts is 45/1,081. Player j then has probability $8 \cdot 7/45 \cdot 44$ of being dealt two hearts. Multiplying then yields

$$P(ij) = \frac{1,260}{1,070,190} = \frac{14}{11,891}.$$

We now move to three players. Let $P(ijk)$ be the probability that three fixed players i, j, k have each been dealt two hearts. Given that players i and j have been dealt two hearts, we have a probability of $6 \cdot 5/43 \cdot 42$ that player k also receives two hearts. Multiplying this by $P(ij)$ yields

$$P(ijk) = \frac{30 \cdot 14}{11891 \cdot 43 \cdot 42} = \frac{10}{511,313}.$$

We consider four players next and let $P(ijk\ell)$ denote the probability that four fixed players are dealt two hearts. We continue as before, that is, multiply

$P(ijk)$ by the probability player ℓ receives two hearts given that the other three players also have hearts. The latter probability is $3/410$. This gives us

$$P(ijk\ell) = \frac{3 \cdot 10}{410 \cdot 511313} = \frac{3}{20,963,833}.$$

The maximum number of players who can simultaneously have hearts is five. Let $P(ijk\ell m)$ be the probability that five fixed players all receive two hearts. The probability that the fifth player also gets two hearts is $1/(39 \cdot 19)$. We multiply this by $P(ijk\ell)$ and have

$$P(ijk\ell m) = \frac{1}{\binom{47}{10}} = \frac{1}{5,178,066,751}.$$

By the principle of inclusion-exclusion for probabilities, we then obtain that the probability no one is dealt two hearts is

$$1 - 10P(i) + 45P(ij) - 120P(ijk) + 210P(ijk\ell) - 252P(ijk\ell m) = .63438$$

so that the probability at least one player is dealt a flush is .36562, or slightly more than one in every three times the situation occurs.

The problem is much easier for four suited cards being on board. If the board has four hearts, then the only way we can avoid a flush being held by at least one player is for all 20 cards chosen randomly from the 47 remaining cards not to include a heart. The total number of ways to choose 20 cards from 47 is $\binom{47}{20}$, and the number of ways to choose 20 cards without a heart among them is $\binom{38}{20}$. So the probability of no flush is

$$\frac{\binom{38}{20}}{\binom{47}{20}} = .003439.$$

Hence, the probability that at least one heart is dealt is .9966.

The above calculations are what a neutral observer, who has no additional information about cards held by any of the players, would observe. But the more interesting situation is the conditional probability for a player involved in the hand who has information on two additional cards, namely, her own hand. Let's take a look at how a little additional information changes the situation.

The first scenario is that there are three hearts on board and she has one heart in her hand. We now follow the same procedure we did above except that now at most four other players may hold two hearts since there are only nine unseen hearts.

We introduce another way to calculate the probability that some number of players all are dealt hearts. Suppose you have k players and are choosing their hands from m cards with r hearts available. There are then $\binom{m}{2k}$ ways to choose the $2k$ cards to be dealt to them, $(2k-1)!!$ ways to partition them into 2-card hands, and $k!$ ways to distribute the hands. On the other hand, there are $\binom{r}{2k}$

ways to choose $2k$ hearts to be dealt to them, $(2k-1)!!$ ways to partition the $2k$ hearts into 2-card hands, and $k!$ ways to distribute the hands. So when we go to calculate the probability, the $(2k-1)!!$ and $k!$ terms both cancel leaving only the quotient of $\binom{r}{2k}$ and $\binom{m}{2k}$. This is the figure we use consistently below.

Label the other players $1, 2, \ldots, 9$ and let $P(i)$ denote the probability that player i has been dealt two hearts. We see easily that

$$P(i) = \frac{9 \cdot 8}{45 \cdot 44} = \frac{2}{55}.$$

Let $P(ij)$ denote the probability that players i, j have been dealt two hearts. The consideration is as above. Hence,

$$P(ij) = \frac{\binom{9}{4}}{\binom{45}{4}} = \frac{2}{2,365}.$$

In a similar fashion, we obtain

$$P(ijk) = \frac{\binom{9}{6}}{\binom{45}{6}} = \frac{1}{96,965},$$

and

$$P(ijk\ell) = \frac{\binom{9}{8}}{\binom{45}{8}} = \frac{1}{23,950,355}.$$

We now employ inclusion-exclusion for probabilities and obtain the probability that no one has a flush to be

$$1 - 9P(i) + 36P(ij) - 84P(ijk) + 126P(ijk\ell) = .70231,$$

so that we have a probability of .2977 that at least one opponent has a a flush.

The second scenario we examine is that a player has no hearts with three hearts on board. What does this do to the probability of one of nine random opponents having a flush? This looks just like the preceding scenario except we now are choosing hearts from 10 hearts, and it is again possible, though unlikely, that five other players have a flush. This time we have

$$P(i) = \frac{90}{45 \cdot 44} = \frac{1}{22}.$$

Continuing, we see that

$$P(ij) = \frac{\binom{10}{4}}{\binom{45}{4}} = \frac{2}{1,419},$$

$$P(ijk) = \frac{\binom{10}{6}}{\binom{45}{6}} = \frac{1}{38,786},$$

$$P(ijk\ell) = \frac{\binom{10}{8}}{\binom{45}{8}} = \frac{1}{4,790,071}, \text{ and}$$

$$P(ijk\ell m) = \frac{\binom{10}{10}}{\binom{45}{10}} = \frac{1}{3,190,187,286}.$$

This time the probability that no one has a flush is

$$1 - 9P(i) + 36P(ij) - 84P(ijk) + 126P(ijk\ell) - 126P(ijk\ell m) = .63951$$

which implies the probability that someone has a flush is .36049.

The third scenario we examine is that a player has two hearts with three hearts on board. This looks just like the preceding two scenarios except we now are choosing hearts from 8 hearts. This gives

$$P(i) = \frac{56}{45 \cdot 44} = \frac{14}{495}.$$

Continuing, we see that

$$P(ij) = \frac{\binom{8}{4}}{\binom{45}{4}} = \frac{2}{4,257},$$

$$P(ijk) = \frac{\binom{8}{6}}{\binom{45}{6}} = \frac{1}{290,895}, \text{ and}$$

$$P(ijk\ell) = \frac{\binom{10}{8}}{\binom{45}{8}} = \frac{1}{8,145,060}.$$

The probability that none of the other 9 hands have two hearts is

$$1 - 9P(i) + 36P(ij) - 84P(ijk) + 126P(ijk\ell) = .76209$$

giving us a probability of .2379 that one or more of them has two hearts.

If the player has no hearts and there are 4 on board, then the probability none of the other 9 hands has a heart is

$$\frac{\binom{36}{18}}{\binom{45}{18}} = .0052888$$

so that the probability someone has a heart is .9947.

If the player has one heart and there are 4 on board, then the probability none of the other 9 hands has a heart is

$$\frac{\binom{37}{18}}{\binom{45}{18}} = .010299$$

so that the probability someone has a heart is .9897.

Fianlly, if the player has two hearts and there are 4 on board, then the probability none of the other 9 hands has a heart is

$$\frac{\binom{38}{18}}{\binom{45}{18}} = .019569$$

so that the probability someone else has a heart is .9804.

We now give a table with the conditional probabilities for a given player in hold'em.

Suited on Board	Suited for Player		
	0	1	2
3	.3605	.2977	.2379
4	.9947	.9897	.9804

Hold'Em from Player's Viewpoint

This concludes our look at hold'em.

Omaha

Now we shall run through similar calculations for Omaha. The calculations are more difficult now because of the fact players have four cards in their hands rather than two. Another interesting feature of Omaha is that players must use two cards from their hands. Consequently, in Omaha, unlike hold'em, the more suited cards there are on board, the less likely it becomes that someone has a flush.

First, we consider the situation that there are three suited cards on board and two cards not in the suit. As above, for simplicity, we assume the three suited cards are hearts. We look first at the situation that we know nothing about any of the 10 hands.

Label the players $1, 2, \ldots, 10$ and let $P(i)$ denote the probability that player i has at least 2 hearts. Player i can have any of $\binom{47}{4} = 178,365$ hands. We count the number of these hands for which i has at least two hearts. There are $\binom{10}{2} = 45$ ways to choose two hearts and $\binom{37}{2} = 666$ ways to choose two cards that are not hearts. Multiplying gives 29,970 hands for player i with exactly two hearts. There are 120 ways to choose three hearts and 37 ways to choose the last card from the non-hearts. This gives 4,440 hands with exactly three hearts. Finally, there are $\binom{10}{4} = 210$ ways to choose four hearts. Summing these numbers gives 34,620 hands for which player i has a heart flush. This tells us that

$$P(i) = \frac{34,620}{178,365} = \frac{2,308}{11,891}.$$

Now consider the probability $P(ij)$ that players i and j both have heart flushes. Player i can have any of 178,365 hands as determined above. Once i's hand is chosen, player j can have any of $\binom{43}{4} = 123,410$ hands. Multiplying gives 22,012,024,650 ways the two players can be dealt hands.

We need to determine how many of the preceding ways that hands can be dealt to the two players result in both having a flush. As we saw above, there are 29,970 ways for player i to have exactly two hearts. This leaves 8 hearts and 35 non-hearts from which player j receives a hand. Thus, there are $\binom{8}{2}\binom{35}{2} = 16,660$ ways for player j to get exactly two hearts. Similarly, there are $35\binom{8}{3} = 1,960$ ways for player j to get exactly three hearts. Finally, there are $\binom{8}{4} = 70$ ways for player j to get four hearts. Hence, given that player i has been dealt exactly two hearts, there are $16,660 + 1,960 + 70 = 18,690$ ways for player j to have received two or more hearts. Thus, there are

$$29,970 \cdot 18,690 = 560,139,300$$

ways for player i to be dealt exactly two hearts and player j to receive at least two hearts.

From above, we know there are 4,440 hands in which player i has exactly three hearts. Player j now is choosing her hand from 7 hearts and 36 non-hearts. This gives $\binom{7}{2}\binom{36}{2} = 13,230$ ways for j to have exactly two hearts, $36\binom{7}{3} = 1,260$ ways for j to have exactly three hearts, and $\binom{7}{4} = 35$ ways for j to have exactly four hearts. Altogether, player j has 14,525 ways to make a flush given that player i has exactly three hearts. Multiplying gives

$$4,440 \cdot 14,525 = 64,491,000$$

ways for player i to have exactly three hearts and player j to receive at least two hearts.

The last situation is the 210 ways in which player i can be dealt exactly four hearts. Now player j is choosing her hand from 6 hearts and 37 non-hearts. The numbers now become $\binom{6}{2}\binom{37}{2} = 9,990$ ways for player j to have exactly two hearts, $37\binom{6}{3} = 740$ ways for player j to have exactly three hearts, and 15 ways for player j to have four hearts. This gives 10,745 ways for player j to make a flush given that player i has exactly four hearts. We then have

$$210 \cdot 10,745 = 2,256,450$$

hands for which players i and j have heart flushes in this case.

Summing all the numbers gives 626,886,750 ways of dealing hands to players i and j so that both have flushes. Dividing by the total number of ways the two players can be dealt hands given above, yields

$$P(ij) = \frac{597,035}{20,963,833}.$$

Now move to the probability that three players are dealt flushes. We use the notation $P(ijk)$. The number of ways the three players can be dealt hands is simply the product of the number of ways two players can be dealt hands multiplied by $\binom{39}{4}$. Taking this product gives

$$1,810,511,039,487,150$$

ways that hands can be dealt to these particular players.

Now let's break down the subcases of patterns for which all three players have flushes in the way the following computations will make clear. The number of ways in which all three have exactly two hearts is given by using the fact we know there are 29,970 ways player i is dealt exactly two hearts. Given that player i has exactly two hearts, we saw above that there are 16,660 ways player

j has exactly two hearts. Given these two facts, player k is choosing 2 hearts from 6, and choosing 2 non-hearts from 33. This gives him $15 \cdot 528 = 7,920$ ways of choosing two hearts. Multiplying gives us

$$29,970 \cdot 16,660 \cdot 7,920 = 3,954,457,584,000$$

ways of dealing precisely two hearts to each of the players.

Keeping everything the same in the preceding paragraph, there are $33\binom{6}{3} = 660$ ways for player k to have exactly three hearts. This gives us a pattern of 2,2,3 for the number of hearts in the three hands. Note that the player receiving three hearts could be any of the three players, so that we multiply by 3 to obtain the number of ways of dealing two players two hearts and one player three hearts. This gives us

$$3 \cdot 29,970 \cdot 16,660 \cdot 660 = 988,614,396,000$$

ways for doing it.

Repeating the same thing except for changing player k getting four hearts in 15 ways, we obtain

$$3 \cdot 29,970 \cdot 16,660 \cdot 15 = 22,468,509,000$$

as the number of ways of dealing two hearts to each of two players and four hearts to the other player.

We now have taken care of 2,2,2 and 2,2,3 and 2,2,4 patterns for the distributions of hearts to the three players i, j, k. Now let's move to patterns with a single 2. We look at 2,3,3 first. There are 29,970 ways for player i to get exactly two hearts. Given that i has two hearts, we saw above that there are 1,960 ways for player j to get exactly three hearts. Player k is now choosing 3 hearts from 5, which can be done in 10 ways, and 1 card from 34. This produces 340 ways for k to get three hearts. Since any of the three players could be the one getting two hearts, we multiply by 3 to obtain

$$3 \cdot 29,970 \cdot 1,960 \cdot 340 = 59,916,024,000$$

ways of giving flushes to i, j, k such that one player gets two hearts and the others get three.

For the 2,3,4 pattern, we multiply by 6 because there are 6 different ways of assigning the values to the distinct players. The other difference from the preceding case is that we are choosing 4 hearts from 5 for player k which can be done in 5 ways. The number we get in this case is

$$6 \cdot 29,970 \cdot 1,960 \cdot 5 = 1,762,236,000.$$

The last pattern involving a 2 is 2,4,4. Here we have only 70 choices for player i following the 29,970 choices for player i. Note that player k has only one choice. We multiply by 3 because any of the three players could be receiving exactly two hearts. Hence, we have

$$3 \cdot 29,970 \cdot 70 = 6,293,700$$

ways of dealing hands to the three players so that one player gets two hearts and the other two get 4.

This brings us to patterns with no 2. The first we consider is 3,3,3. We know there are 4,440 ways for player i to get exactly three hearts and 1,260 ways for j also to get three hearts. Player k then is choosing 3 hearts from 4, and choosing 1 card from 35. This can be done in 140 ways. Thus, there are

$$4,440 \cdot 1,260 \cdot 140 = 783,216,000$$

ways of dealing three hearts to each of the players.

The last pattern is 3,3,4. We multiply by 3 since any of the three players can have four hearts. This gives us

$$3 \cdot 4,440 \cdot 1,260 = 16,783,200$$

choices because the last player has no choice but to use the remaining four hearts.

We now sum all the appropriate numbers above and obtain 5,028,025,041,900 of the deals to the three players with three flushes. Dividing by the total number of deals gives

$$P(ijk) = \frac{14,380,166}{5,178,066,751}$$

for the probability of the three given players all receiving flushes.

This brings us to $P(ijk\ell)$, the probability that four players have been dealt flushes. The total number of ways for four players to be dealt four cards each is $\frac{47!}{(4!)^4 31!} = 94,798,358,027,547,174,000$.

Since there are four players and only 10 cards to be spread among them, there are not so many patterns to consider. One possibility is that we have a 2,2,2,2 pattern of hearts. Many of these numbers already have been worked out above. There are 29,970 ways for player i to get two hearts, 16,660 ways for player j to get two hearts, and 7,920 ways for player k to get two hearts. Player ℓ is choosing two hearts from the four remaining hearts, which can be done in 6 ways, and two non-hearts from the remaining 31, which can be done in 465 ways. Thus, player ℓ can be dealt two hearts in 2,790 ways. Multiplying all these numbers gives 11,032,936,659,360,000 possibilities.

For the pattern 2,2,2,3, we must multiply by 4 because any one of the players may receive three heaerts. The numbers remain the same as the preceding paragraph except that there are 4 ways for player ℓ to receive three hearts from the remaining four. She then has 31 choices for the last card. This gives 124 hands for player ℓ. The product we get here is then 1,961,410,961,664,000.

For the pattern 2,2,2,4, we again must multiply by 4, but now player ℓ has no choice. The product this time is 15,817,830,336,000.

The last pattern is 2,2,3,3 for which we must multiply by 6. For player i, there are 29,970 hands again, and for player j there are 16,660 hands. We saw above that there are then 660 ways for player k to have three hearts. Player ℓ has no choice for her three hearts, but does have 32 choices for the non-heart.

Multiplying these numbers produces 63,271,321,344,000 deals for the pattern 2,2,3,3.

Summing the values gives us 13,073,436,772,704,000 deals altogether. Dividing by the total number of hands that can be dealt to these four players produces

$$P(ijk\ell) = \frac{37,584}{272,529,829}.$$

The last probability we have to work out is for five players being dealt flushes. Since there are only 10 hearts available, the only possible pattern is that each of the players is dealt two hearts. The total number of hands that can be dealt to five players is $\frac{47!}{(4!)^5 27!} = 2,982,830,335,336,771,829,910,000$. There are then 29,970 hands for player i, 16,660 hands for player j, 7,920 ways for player k to get two hearts, and 2,790 ways for player ℓ to be dealt two hearts. Player m has no choice for the two hearts, but is choosing two cards from 29 for the non-hearts. This gives 406 choices for her hand. Multiplying all these numbers gives 4,479,372,283,700,160,000 ways for all five to receive two hearts. Dividing by the total number of ways yields

$$P(ijk\ell m) = \frac{7,776}{5,178,066,751}$$

as the probability five players are dealt flushes.

We now use inclusion-exclusion to obtain the probability that nobody is dealt a flush. This is given by

$$1 - 10P(i) + 45P(ij) - 120P(ijk) + 210P(ijk\ell) - 252P(ijk\ell m) = .03593$$

which is close to 1. Thus, we obtain a probability of .964 that at least one random hand has a flush.

We now perform the same calculations for four suited cards on board but skip the description. We simply list the probabilities so that people can check them independently. Note that it is now impossible for five people to have a flush because there are only nine hearts amongst the remaining 47 cards.

1. $P(i) = \frac{9,542}{59,455}$
2. $P(ij) = \frac{1,918,243}{104,819,165}$
3. $P(ijk) = \frac{1,684,828}{1,362,649,145}$
4. $P(ijk\ell) = \frac{43,632}{1,362,649,145}$

Using inclusion-exclusion, we have

$$1 - 10P(i) + 45P(ij) - 120P(ijk) + 210P(ijk\ell) = .07696$$

as the probability that none of the random hands is dealt a flush. Thus, there is a probability of .923 that at least one random hand out of ten has a flush, when the board has four suited cards.

We now repeat the computations for five suited cards on board. We use the same notation as above, but this should not cause any problems since the context is clear.

1. $P(i) = \frac{23{,}002}{178{,}365}$
2. $P(ij) = \frac{310{,}979}{28{,}587{,}045}$
3. $P(ijk) = \frac{48{,}012}{104{,}819{,}165}$
4. $P(ijk\ell) = \frac{432}{104{,}819{,}165}$

Using inclusion-exclusion, we have

$$1 - 10P(i) + 45P(ij) - 120P(ijk) + 210P(ijk\ell) = .1458$$

as the probability that no one has a flush with five suited cards on board. Thus, there is a probability of .8542 that at least one random hand out of ten has a flush, when the board has five suited cards.

We now give a table listing the probabilities we have calculated for both hold'em and Omaha with a neutral observer and ten random hands. The

Suited cards on board	Hold'Em	Omaha
3	.3656	.964
4	.9966	.923
5	1	.8542

Neutral Observer

entry 1 in the table indicates that everyone has a flush when there are five suited cards on board in hold'em.

Now we are going to look at the conditional probabilities for a fixed player. Since there are several parameters involved here, we are going to introduce some extra notation. Indeed, we shall let $P(a, b; i)$ denote the probability that player i has a flush given that there are a hearts on board and a fixed player with b hearts, where i runs through the other nine players. Similarly, $P(a, b; ij)$ means that players i, j have flushes under the same conditions on the board and our fixed player with b hearts. We first do the cases for which our given player has a flush too.

We first consider $a = 3, b = 2$. This leaves 8 hearts unaccounted for. We have

1. $P(3, 2; i) = \frac{267}{1{,}763}$

2. $P(3,2;ij) = \frac{246{,}229}{16{,}112{,}057}$

3. $P(3,2;ijk) = \frac{668}{848{,}003}$

4. $P(3,2;ijk\ell) = \frac{144}{16{,}112{,}057}$

Inclusion-exclusion gives a probability of .8779 that at least one hand out of the other nine random hands also has a flush.

We now consider $a = 3, b = 3$. This leaves 7 hearts unaccounted for. We have

1. $P(3,3;i) = \frac{415}{3{,}526}$

2. $P(3,3;ij) = \frac{132{,}584}{16{,}112{,}057}$

3. $P(3,3;ijk) = \frac{3{,}564}{16{,}112{,}057}$

Inclusion-exclusion gives a probability of .7816 that at least one hand out of the other nine random hands also has a flush.

Next we look at $a = 3, b = 4$. This leaves 6 hearts unaccounted for. We have

1. $P(3,4;i) = \frac{307}{3{,}526}$

2. $P(3,4;ij) = \frac{1{,}652}{435{,}461}$

3. $P(3,4;ijk) = \frac{108}{3{,}048{,}227}$

Inclusion-exclusion gives a probability of .65 that at least one hand out of the other nine random hands also has a flush.

We increase the value of a and consider $a = 4, b = 2$. This leaves 7 hearts unaccounted for and all parts of the probability computation are the same as for $a = 3, b = 3$ giving us a probability of .7816 in this case. Similarly, when $a = 4, b = 3$, we obtain .65 that at least one of the other nine random hands has a flush.

The first new value is for $a = 4, b = 4$. This leaves only 5 hearts in the unseen cards. We then obtain

1. $P(4,4;i) = \frac{1{,}483}{24{,}682}$

2. $P(4,4;ij) = \frac{218}{160{,}433}$

Inclusion-exclusion gives a probability of .4918 that at least one hand out of the other nine random hands also has a flush.

The value of $a = 5, b = 2$ is the same as for $a = 3, b = 4$, and the value for $a = 5, b = 3$ is the same as for $a = 4, b = 4$. This leaves $a = 5, b = 4$ in which case there are only 4 hearts left in the deck. For this case we have

1. $P(5,4;i) = \frac{4{,}603}{123{,}410}$

2. $P(5, 4; ij) = \frac{18}{61{,}705}$

Inclusion-exclusion gives a probability of .3252 that at least one hand out of the other nine random hands also has a flush.

We now move to the cases that our given player does not have a flush, that is, when $b \in \{0, 1\}$. When $a = 3, b = 0$, there are 10 hearts and 33 non-hearts comprising the 43 cards left over. This case was not done before and we follow the same procedure.

1. $P(3, 0; i) = \frac{399}{1{,}763}$
2. $P(3, 0; ij) = \frac{634{,}749}{16{,}112{,}057}$
3. $P(3, 0; ijk) = \frac{1{,}275{,}562}{273{,}904{,}969}$
4. $P(3, 0; ijk\ell) = \frac{552{,}528}{1{,}917{,}334{,}783}$
5. $P(3, 0; ijk\ell m) = \frac{7{,}776}{1{,}917{,}334{,}783}$

Inclusion-exclusion gives a probability of .974 that at least one hand out of the other nine random hands has a flush.

We conclude with the two cases of $a = 3, b = 1$ and $a = 4, b = 0$. There are 9 hearts and 34 non-hearts comprising the 43 cards left over. We do the specific calculations for one of them and note that the entry in the table below is the same for both cases.

1. $P(3, 1; i) = \frac{11{,}589}{61{,}705}$
2. $P(3, 1; ij) = \frac{2{,}055{,}441}{80{,}560{,}285}$
3. $P(3, 1; ijk) = \frac{1{,}184{,}284}{563{,}921{,}995}$
4. $P(3, 1; ijk\ell) = \frac{38{,}448}{563{,}921{,}995}$

Inclusion-exclusion gives a probability of .9396 that at least one hand out of the other nine random hands has a flush.

We now present the latter results for these conditional probabilities as a table. The entry is the probability at least one player out of 9 random Omaha hands has a flush under the conditions of the table.

Suited on board	Suited for player				
	0	1	2	3	4
3	.974	.9396	.8779	.7816	.65
4	.9396	.8779	.7816	.65	.4918
5	.8779	.7816	.65	.4918	.3252