Full 6-Card Hands

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Abstract

We count the number of different kinds of full 6-card hands.

1 Introduction

The file entitled "6-Card Poker hands" ranks the hands based on the best 5-card hand contained in it. This file is doing something different, namely, we are ranking 6-card hands based on all six cards. Consequently, there are some hands that do not exist when considering just 5 cards. For example, we now have a hand consisting of two sets of 3-of-a-kinds. We call this hand a *double trips* hand. There also is a hand that consists of 4-of-a-kind and a pair. We call that hand a *full motel* hand.

The total number of hands under discussion is given by

$$\binom{52}{6} = 20,358,520. \tag{1}$$

2 Calculations

We are going to partition the 6-card hands according to the number of ranks present. This makes it easy to verify that we have included all possibilities.

2.1 Two Ranks

The two patterns for sets of six cards with only two ranks present are xxxxyy and xxxyyy. The number of the first type is given by $13 \cdot 12 \cdot 6 = 936$. Hence, there are 936 full motels.

There are $\binom{13}{2}4^2 = 1,248$ hands of the form xxxyyy. Hence, there are this many double trips hands. These values are entered in the table below.

2.2 Three Ranks

Hands with three ranks have one of the three forms xxxxyz, xxxyyz or xxyyzz. The first hand is 4-of-a-kind and there are 13 choices for the rank x, $\binom{12}{2} = 66$

choices for the other two ranks, and 4 choices for each card of the two given ranks. Multiplying yields 13,726 4-of-a-kind hands.

For full houses there are 13 choices for the rank of the trips, 12 choices for the rank of the pair, and 11 choices for the rank of the singleton. There are 4 choices for the trips of rank x, 6 choices for the pair of rank y, and 4 choices for the singleton of rank z. Multiplication produces 164,736 full houses.

For the hand with three pairs, there are $\binom{13}{3} = 286$ choices for the three ranks and 6 choices for a pair of each rank. We obtain $286 \cdot 6^3 = 61,776$ three pairs hands.

2.3 Four Ranks

There are two patterns for hands with four ranks, namely, xxxyzw and xxyyzw. There $13\binom{12}{3}4^4 = 732,160$ hands of the first type.

The number of ways of holding two pairs is given by choosing two ranks from 13 for the pair ranks, two ranks from 11 for the singleton ranks, 6 ways of choosing each pair, and 4 ways of choosing each singleton. This gives $78 \cdot 55 \cot 36 \cdot 16 = 2,471,040$.

2.4 Five Ranks

The only pattern for five ranks is xxyzws. There are 13 choices for the rank x, 6 choices for a pair of that rank, $\binom{12,4}{=}495$ choices for the other four ranks, and 4 choices for a card of each of the ranks. Multiplying yields $78 \cdot 495 \cdot 4^4 = 9,884,160$ hands with a single pair.

2.5 Six Ranks

Six ranks is interesting because this now allows straights and flushes. First think of rank sets. There are $\binom{13}{6} = 1,716$ possible sets with 6 distinct ranks. Of these, 9 consist of 6 consecutive ranks and the remaining 1,707 are not consecutive.

A rank set with 6 consecutive ranks allows $4^6 = 4,096$ choices for the suits of the various ranks. Of these choices, exactly 4 have all six cards in the same suit. The other 4,092 choices produce straights. Thus, there are 4 royal flushes, 32 straight flushes, and $9 \cdot 4092 = 36,828$ straights.

The other 1,707 rank sets produce $4 \cdot 1707 = 6,828$ flushes and $4092 \cdot 1707 = 6,985,044$ high card hands.

3 Table

Here is a table containing all the values obtained above.

hand	number	probability
royal flush	4	.0000002
straight flush	32	.000002
full motel	936	.00046
double trips	1,248	.000061
flushe	6,828	.000354
4-of-a-kind	13,728	.000674
straight	36,828	.00181
three pairs	61,776	.00303
full house	164,736	.00809
3-of-a-kind	732,160	.03596
two pairs	2,471,040	.12138
high card	6,985,044	.3431
one pair	9,884,160	.4855

Some comments are in order. Note that flushes and straights are now stronger than for 5-card games because getting six suited cards out of six cards is rare as is making a 6-card straight. A flush is better than 4-of-a-kind and a straight is better than a full house. Another feature that jumps out at us is that it is harder to get a hand that is just a high card hand compared to a pair. Thus, it must be ranked higher.

Thus, one must exert some care if one is going to build a game based on six cards where all six cards are used.