Diocles And The Earliest Extant Discussion of Gnomonics

J. Len Berggren (Burnaby, B.C. Canada)

Greek sundials are fascinating for much the same reason that ancient Greek society is fascinating, namely that, although it is so different from the modern, it is, in many ways, the foundation for modern society. Among the differences (gnomonically speaking) are that Greek sundials told not the equal hours of the day we know, but the seasonal hours, in which each of the periods of daylight and night was divided into twelve hours. (The lengths of these naturally differ between winter and summer.) In addition, Greek sundials indicated the hour by the shadow of the tip of the gnomon, not its edge.[3] And, the most common modern dial, with its gnomon parallel to the N-S axis of the earth, was unknown to the Greeks. Yet, despite these differences, it is not too much to say that our modern approach to dials, based on mathematics and astronomy, originated with the Greeks.

Greek sundials are also fascinating because the time and circumstances of their origins are so obscure. Thus, although one finds ambiguous literary references to shadow casting instruments as early as the historian Herodotus in the fifth century BCE, the oldest existing ancient dials belong to the third century BCE. And, other than the vague references by Herodotus and others, credible literary evidence begins in that century as well, that is during the lifetime of Archimedes, who was killed by a Roman soldier in 212 BCE.

In fact, shortly after Archimedes' death, a geometer named Diocles wrote a treatise *On Burning Mirrors,* devices which reflect the sun's rays to a point so as to cause burning.[4] Since this treatise contains the earliest discussion of problems in gnomonics, it will, I hope, be of interest.

In his *Burning Mirrors,* Diocles discusses a number of geometrical problems, including two that concern the reflection of solar rays.[5] These are:

1. To find a mirror surface such that when it is placed facing the sun, the rays are reflected onto the circumference of a circle; and
2. To find a mirror surface such that, when it is placed facing the sun, the rays are reflected to a single point.

Diocles goes on to tell us that a man named Dositheus, whom we know as a correspondent of Archimedes, solved the second.[6] As for the first problem, he says that "since it was only theoretical, and there was no argument worthy to make it known, [it] was not solved practically. We have," he continues, "set out a compilation of the proofs of both these problems and elucidated them."[7] Later he says, "an ingenious method has been found for a burning mirror to burn without being turned to face the sun,"[8] but makes no claim that the discovery was his. It seems, then, that Diocles claims as his own only the organization and exposition of the results of others.

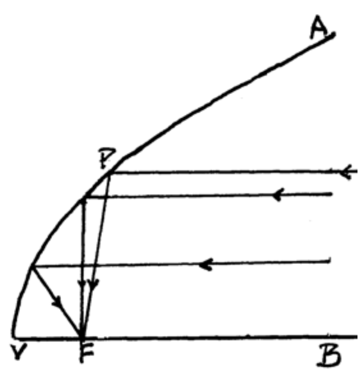

Figure 1.

The solution to the first problem depends on the solution to the second, which employs the focus of a parabola. Therefore, I shall spend just a minute on the second problem, reflecting the rays of the sun to a single point. Thus, suppose a concave mirror is formed in the shape of a surface created when half a parabola, the line APV, with vertex V is rotated about its axis VB. (See Figure 1, adapted from Diocles 1976). Then the concave parabolic surface of the mirror will, if it is pointed toward the sun, reflect all the incoming rays so that they pass through the focus F on its axis. This creates tremendous heat at F, sufficient to cause burning.

With this as background we may now turn to Problem 1, which is directly relevant to sundials. About it Diocles writes (translation slightly modernized): “The problem ...is also solved by a parabola being revolved ...(Thus) an ingenious method has been found for a burning-mirror to burn without being turned to face the sun: instead it is fixed in one and the same position, and indicates the hours of the day without a gnomon. It does this by burning the place to which the rays are reflected: their reflection is always the place for the position of the hour, which is sought. This statement is amazing, namely that there is no need to turn the mirror at all but that (what we have described) results merely from the above-mentioned figure.”[9]

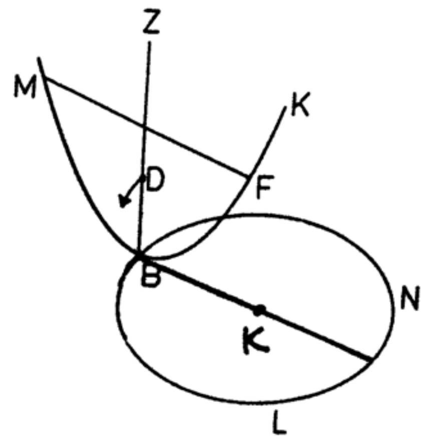

Figure 2.

Problem 1 actually has two versions, both of which Diocles discusses. In the first version the problem is to construct a mirror surface such that the rays are at each moment reflected to the whole of the circumference of a circle (or at least to some arc of the circumference). Diocles' solution to the first version is as follows: In Fig. 2 (adapted from Hogendijk 1985) let MBF be a section of a parabola in a plane perpendicular to the plane of the circle BLN of center K. Let the plane of the parabola be so positioned that it passes through the center of circle BLN. Now revolve the curve MB around the circumference of the circle so that its plane always passes through the center of the circle and it always stays perpendicular to the circle. Then we get a surface as in Fig. 3 (from Hogendijk 1985). Diocles' argument here is not very clear. But it seems to come down to the claim that since the incoming solar rays hit points of parabolas, their reflections pass through the circle that the focus, D, of the parabola traces out as the parabola is rotated.

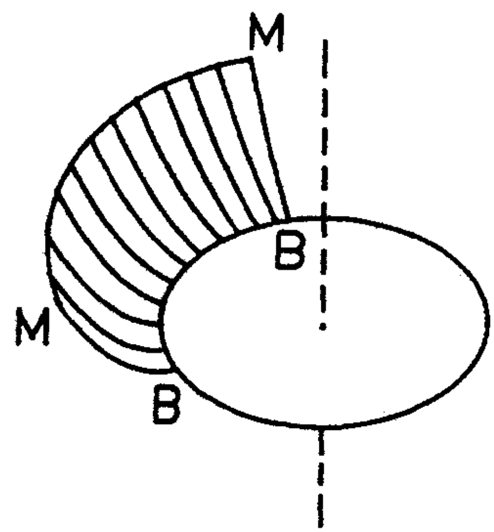

Figure 3.

It is a neat idea, and one that solves Pythion's problem. Indeed, whenever the device is turned to face the sun the incoming rays striking the parabolic sections that make up the surface are parallel to the axes of the parabolas they strike, and so are reflected to the foci.

Incidentally, Diocles says that “And not only is it possible for reflection to occur to the circumference of a circle, but it might also be possible for it to occur to any other curve we wish.”[10] Certainly, Greek geometry was in no position to handle arbitrary curves, and Diocles' discussion is no more than a somewhat confused explanation of how to generalize the case of the circle to an arbitrary curve.[11] However, the Hellenistic geometer who discovered the device deserves credit for seeing the possibility of generalizing the solution from circles to more general curves. And, according to Hogendijk, a proof at least for the ellipse, hyperbola, and parabola would have been possible for the best geometers of the time.[12]

In any case, the second version of Problem 1 is more relevant to sundials. This problem requires that one construct a device that, once mounted in position, reflects the sun's rays onto successive points of a circle at successive times of the day. Diocles' solution is to revolve a section of a parabola, MBF in Figure 4 (taken from Hogendijk 1984) by 90° on both sides around a line MF perpendicular to its axis. This produces the surface shown in Figure 4. In this surface, the sections by planes containing line MEF are sections of parabolas and the sections by planes perpendicular to the line MEF are circular arcs.

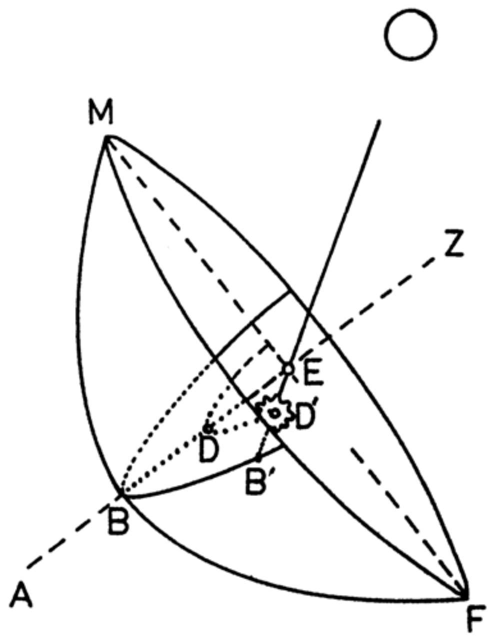

Figure 4.

Will the device that Diocles described work? It is easy to see that if one is to have any hope of it working one must mount the device so that the axis MEF is parallel to the celestial N-S axis.[13] This means that the plane BB'E is the plane of the equator. Then, when the sun is on the equator, *i.e.* it is at one of the equinoxes, its rays when it is opposite B' will be parallel to the axis EB' of the parabola MB'F. Hence, all rays reflected from that infinitely thin parabolic section MB'F of the mirror will focus at O'.

But there is more. As we remarked above, not only is any section of the surface by a plane containing the axis BZ a parabola, but also any section by a plane perpendicular to the line MEF will be a circle. And, elsewhere in his treatise, Diocles demonstrates a proposition that implies that if the distance BE on the axis is chosen appropriately then in fact the rays reflected from a circular arc of 30° on either side of B' fall onto a small piece of the line B'E near D'.[14]

Diocles no doubt felt - correctly as it turns out - that from these two facts it follows that a ray hitting any part of the surface of the mirror near B' will be reflected to a place near to D'.[15] Hence one could argue that, if not burning, at least some sort of intense heat would be produced near D.'[16]

Another point of interest in Diocles' treatise is his statement that, "As for the matter of the gnomons used by the astronomers, they a have great accuracy when they are made according to the old methods which used to be employed in making time-measuring instruments in which the shadow is used. But, many of the surfaces in which it (the shadow) is used are impossible to make, and many of them are very difficult to make. In short you must realize that the knowledge of this is difficult and requires care and perseverance; whoever has spent pains on it will attest the truth of what we say."[17] It is not clear what surfaces Diocles is referring to here. The most common surviving non-planar dials from antiquity use portions of the concave surfaces of spheres or cones. And since the surface of a conical dial can be generated by a rotating straight line, while this is not true of a hemispherical dial, the conical dials would seem to be easier to make. And, in fact, these are the most common type of the surviving ancient dials.[18]

Diocles also leaves us with a few puzzles about ancient sundial theory. One stems from his remark that, "But those time-measuring instruments using the shadow which indicate the hours without having a gnomon in them reach a degree of minute accuracy such as cannot be attained in this matter by any other kind (of instruments)." Were these the roofed spherical dials Gibbs describes in her book, which indicated the time by a spot of sunlight? If so, one needs to reconsider her suggestion[19] that "the roofed spherical dial, with the earliest known example dating from the first century B.C., was probably one of the latest contributions to Graeco-Roman dialing," since Diocles lived well before that time.

Finally, I put a challenge to our mechanically skilled members: Make a dial according to Diocles' specifications and see what sort of heat is indeed created in the vicinity of the path of the focus of the generating parabola.

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J. Len Berggren, Simon Fraser University, Burnaby, B.C. Canada V5A 1S6 berggren@sfu.ca

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[3] Gibbs [1976,4] points out these two differences.
[4] For Toomer's arguments for dating Diocles to the early second century BCE, see Diocles [1976, I –2].
[5] The first problem was posed by a certain Pythion of Thasos, not otherwise known, and the second by the mathematician and gnomonist Zenodorus, who is known from other sources and seems to have been a contemporary of Diocles. The problems are stated in Diocles [1976,34].
[6] Diocles [1974, 34]. Archimedes' alleged use of burning mirrors to defend his hometown of Syracuse against the Roman fleet has contributed greatly to his fame in ancient and modem times. It is therefore of interest to see here the name not of Archimedes but that of his correspondent Dositheus linked to a proof of the focal property of a parabolic mirror.
[7] Diocles [1976, 34].
[8] Diocles [1976, 38].
[9] Diocles [1976, 36-38].
[10] Diocles [1976, 52].
[11] Hogendijk [1985, 171-73].
[12] Hogendijk [1985, 174].
[13] If this condition is not satisfied, then, however the parabola is mounted, the rays can enter the device parallel to the axis of one of the parabolic sections (such as MB'F) at most once during the day. And only then can the reflected rays focus onto the point D'.
[14] This and the following observation are due to Hogendijk [1985, 176-78].
[15] For an intuitive argument supporting this, as well as for a rigorous proof in the ancient style, see Hogendijk [1985, 178-81].
[16] In the discussion subsequent to this talk at the NASS Tucson conference, Larry McDavid of Anaheim, CA, pointed out that the idea of focusing the sun's rays to cause burning (or, at least, scorching, along a path, for the purpose of measuring the number of hours of sunlight in a day is the basis of the Campbell-Stokes sunshine recorder, a device invented in the nineteenth century. For details of this and other sunshine recorders see Middleton [1976, 231-244].
[17] Diocles [1976, 42].
[18] According to Gibbs [1976, 66 and 73] there are 98 known surviving ancient dials with spherical faces and 109 with conical faces. Naturally, the difference is hardly significant, given the slight difference in numbers and the vicissitudes of time.
[19] Gibbs [1976, 64]