Sundials in Medieval Islamic Science and Civilization J. L. Berggren (Simon Fraser University, Burnaby, B.C. Canada)

Sundials were ubiquitous in medieval Islam, since their shadows showed when it was time to perform the midday and afternoon prayers, two of the five prayers the Prophet Muhammad had prescribed for faithful Muslims.¹ How these two daytime prayer times were defined in early Islam is still an open question, but by the eighth century of our era they had come to be defined by the increase of the shadow over its minimum at midday. Thus, the midday prayer began just after the noon shadow crossed the meridian, or when the shadow had been observed to increase, or (in medieval Andalusia and North Africa) when it had increased by 1/4 the length of the gnomon. The afternoon prayer began when the shadow of a vertical gnomon exceeded the length of its noon shadow by the length of the gnomon, and it ended either at sunset or when the excess of the shadow over the noon shadow was twice the length of the gnomon. This definition of the time of the afternoon prayer was inspired by a simple, popular formula relating the time in seasonal hours to shadow increases.² The idea was to perform the midday prayer when six hours of daylight had passed 3 and the afternoon prayer was exactly between noon and sunset, so that nine seasonal hours of daylight had passed since dawn.⁴ Once the daytime prayers were defined by shadows, the sundial was a most appropriate device for regulating them.

One possible reason that sundials became the instrument of choice for regulating prayer times was that many of the lands conquered by Islam were once part of the Hellenistic or Roman world, where there was a strong tradition of sundials. How common these dials were in antiquity is reflected by the joke from the *Greek Anthology*: "If you put your nose pointing to the sun, and

³ Seven in the case of the Andalusian and North African definition.

⁴David A. King *Islamic Astronomical Instruments*. London: Variorum Reprints, 1987 [hereinafter *IAI*]. Paper XVIII, Appendix B. open your mouth wide/ You will show all passersby the time of day."⁵ The Arab conquerors would have encountered sundials everywhere they went and could hardly fail to have been impressed by a device that could, by means of a shadow, tell how many hours of the day had passed or how many remained until sunset, the beginning of the Muslim day. Around the year 700, as early as 70 years after the beginning of the Muslim era, the Umayyad Caliph Abd al-Aziz, ruling in Damascus, used what would undoubtedly have been a Graeco-Roman sundial for regulating the times of his prayers by the seasonal hours.⁶

Coupled with this visual impression was the Muslim scientists' high regard for their Greek predecessors. In particular they were attracted by Greek trigonometry (plane and spherical) and methods of projecting the surface of the threedimensional celestial sphere onto a plane. There is no direct evidence that Ptolemy's gnomonic treatise The Analemma,' in which a highly sophisticated projection method was described, was translated into Arabic, but we do know that another treatise, also called The Analemma and written by one Diodoros, was translated into Arabic. The Persian polymath, al-Biruni of Ghazna (now in Afghanistan), the greatest scientist of the medieval period, Christian or Islamic, quotes it in his magisterial treatise, On Shadows.⁸ (Diodoros himself is addressed in the aforementioned Greek Anthology as: "Diodoros,

⁷Discussed in F. Sawyer, 'Ptolemaic Coordinate Sundials,' *The Compendium* 5(3) Sept. 1998, 17-24, and G. Ferrari, 'Some Sundials Based On Ptolemaic Coordinates,' *The Compendium* 6(1), March 1999, 9-19.

¹ The other three were to be performed between daybreak and sunrise, at sunset, and at nightfall.

² Seasonal hours were so called because their lengths varied with the seasons, since there were 12 seasonal hours in every period of daylight and 12 in every period of night.

⁵Quoted in Sharon Gibbs, *Greek and Roman Sundials*, New Haven, Conn.: Yale, 1976, 3.

⁶D. A. King, article 'Mikat' in *The Encyclopaedia of Islam*, 7, fac. 115-16. Leiden: E. J. Brill, 1990 (hereinafter El₂). Reprinted in D. A. King, *Astronomy in the Service of Islam*, London: Variorum Reprints, 1993, V.

⁸E.S. Kennedy (trans. and comm.) *The Exhaustive Treatise on Shadows*. Vol. I (translation) and Vol. II (Commentary). Aleppo, Syria: Institute for the History of Arabic Science (1976).

famed among the makers of gnomons, tell me the time!"⁹)

Perhaps because they were eager for chances to show the utility of their subject to the faithful, dozens of Muslim mathematical scientists wrote treatises on the topic, most of them still unstudied. It is also likely that two other factors came into play in motivating the extensive work medieval Islamic work in gnomonics: (1) the challenge of solving difficult problems in mathematical projections, and (2) the opportunity the subject provided for composing tables, an activity for which medieval Muslim scientists had a special passion.¹⁰

Whatever the motivating factors may have been, two of the most famous early Islamic scientists wrote on sundials.

One of these was Habash al-Hasib, who worked in Baghdad in the ninth and tenth centuries and wrote on, among other subjects, horizontal dials with a vertical gnomon of standard length.¹¹ He composed perhaps the first tables for making sundials, tables which recorded for each of ten latitudes, and for each seasonal hour on both of the solstices, the height of the sun over the horizon and the length and direction of the gnomon's shadow.

By the end of the tenth century Muslim scientists had invented the polar dial,¹² the equatorial dial,

¹¹See King, "Al-Khwarizmi and new trends in mathematical astronomy in the ninth century" in *Occasional papers on the Near East*. New York: Hagop Kevorkian Centre, New York University, 2 (1983). David King has informed me that these tables, although attributed to al-Khwarizmi in the sole surviving manuscript, are in fact by Habash.

¹²The single surviving Islamic example of this type was constructed in Acre in 1786-87. The dial is illustrated in King, *IAI*, XVI, Plate 6.

and the horizontal dial with the gnomon parallel to the polar axis.¹³ They had also written, probably as early as the ninth century, the earliest text on portable conical sundials.¹⁴

Another famous writer was Thabit ibn Qurra who worked in the latter half of the 9th century, and came from Harran, a city where books from the library of Alexandria were supposed to have been taken when the Arabs conquered that city.¹⁵ Thabit's work is highly reminiscent of Ptolemy's Analemma and very much in the theoretical mode we associate with the Greek geometers. It was devoted to the transformation of coordinates between celestial coordinate systems based on three planes: the celestial equator, the local horizon, and the plane of the sundial, however that may lie with respect to the first two. The treatise is impressive from a mathematical point of view but in fact most Muslim astronomers were more interested in the practical side of gnomonics.¹⁶

With Thabit, writing on sundials seems to have become a family tradition. However, like certain heriditary traits, that of gnomonophilia skipped a generation, and Thabit's grandson. Ibrahim ibn Sinan, around the tender age of 16, in the early 920s, wrote a treatise treating, under a single principle, all sundials that, up to his time, had been treated separately-horizontal, meridian, and east-west. Only parts of this treatise have survived, but we know what it contained, thanks to the testimony of Ibn al-Haytham, known in the West as Alhazen and famous for his great work, The Optics. Ibn al-Haytham, who wrote about three-quarters of a century after Ibrahim, around the year 1000, tells us, in his Treatise on the Hour Lines, that Ibrahim criticized the Greeks for believing that the hour lines were straight, but,

¹⁵The story that the Arabs burned the books in the library at Alexandria in an excess of religious zeal is a late fabrication, first found in the writings of a 13th-century Muslim historian. (See the article "al-lskandariyya," El₂, IV, Col. 132a.)

¹⁶D. A. King, article 'Mizwala' ('Sundial') in El₂, 7, fac. 115-16, reprinted King, *ASI*, Paper VIII.

⁹Quoted in Thomas L. Heath, *A History of Greek Mathematics*, Oxford: Oxford U. Press, 1923, v. II, 287.

¹⁰There are no known medieval treatises giving geometrical constructions for sundials, according to D.A. King, *IAI*, Paper I ('Astronomical Instrumentation in the Medieval Near East'), p. 10. For overviews of the fruits of this passion for constructing tables consult E. S. Kennedy, *A Survey of Islamic Astronomical Tables*, Transactions of the American Philosophical Society. N.S. vol. 46, pt. 2 (1956), and many of the papers collected in King, *IAI*.

¹³See in King, *IAI*, Paper I ('Astronomical Instrumentation in the Medieval Near East'), 11.

¹⁴D. A. King, article 'Mizwala' ('Sundial') in El₂, 7, fac. 115-16, reprinted in *Astronomy in the Service of Islam*, London: Variorum Reprints, 1993 (hereinafter *ASI*), VIII.

himself, showed only that some of the hour lines were not straight lines.¹⁷ Not only was Ibn al-Haytham able to show this was true for all hours other than noon, but he also showed that each hour line was contained between two straight lines forming a very small angle. Hence, he concluded, they were nearly straight and Ibrahim had been unfair in his critique of the Greeks. (His concern to vindicate the Greeks shows, by the way, the high regard the very first rank of Muslim scientists had for the scientific achievements of the ancient Greeks.) Despite the opinions of Ibrahim ibn Sinan and Ibn al-Haytham, however, the controversy on the hour lines continued, and the thirteenth-century Tunisian astronomer, Ibn al-Raggam, claimed that the hour lines on a horizontal dial are conic sections.

The earliest Islamic treatise on vertical dials is from the tenth century, and an example of such a dial, declining 163° from north, may be found on the wall of the late fifteenth century madrasa (a school, popularly known as the 'Ashrafiyya') of Sultan Qaytbay in Jerusalem. (This dial displayed not only prayer lines but 'prayer warning' lines.) The fact that these dials were particularly suited to display on the walls of mosques and madrasas ensured their popularity in medieval Islam.¹⁸

The earliest surviving Islamic dial comes from eleventh century Cordoba (Fig. 1). It is the work of a well-known astronomer, Ibn al-Saffar, although its crudely executed work, and the kinks in the lines for the third, fourth and eighth hours, as well as in the equinoctial line, do nothing for his reputation either as an astronomer or as a craftsman! The dial displays lines for seasonal hours of the day, lines indicating the beginnings of the seasons, and lines for midday and afternoon prayers. The length of the vertical gnomon is indicated by the radius of a circle inscribed on the plate. (This common feature of Islamic dials provides a sort of built-in repair kit.)

In the context of the religious institutions of mosque and madrasa the religious purpose of the sundials could completely outstrip any secular utility they might have had. Indeed, a fourteenth century Tunisian dial (Fig. 2) showed only the prayer lines and a line indicating the time at which people began to gather at the mosque in preparation for the Friday sermon.¹⁹



Fig. 1 courtesy of Museo Arqueológico Provincial de Córdoba

As the repertoire of dials developed, writers such as the astronomer, al-Marrakushi, composed comprehensive treatments on dialing, in which he discussed horizontal and vertical dials, either cylindrical or conical—as well as simple, portable vertical sundials, in plane, cylindrical or conical formats. Al-Marrakushi, who hailed from the Maghrib, worked in Cairo in the late thirteenth century, and his treatise on astronomical instruments became a standard compendium on the subject. It has an extensive treatment of sundials, complete with tables and diagrams, and the cylindrical dial on which al-Marrakushi wrote was a Muslim invention of the ninth century.²⁰

¹⁷For Ibn al-Haytham's unpublished treatise see the account in Jan P. Hogendijk, 'Le traité d'Ibn al-Haytham sur les lignes horaires,' *Cahier du seminaire Ibn al-Haytham*, 4 (January) 1994: 5-7.

¹⁸This point is made by A.G. Walls and D.A. King in, *IAI*, Paper XVII ('The Sundial on the West Wall of the Madrasa of Sultan Qaytbay in Jerusalem'), 18, where a photograph of the dial is published.

¹⁹King, *IAI*, Paper XVIII ('A Fourteenth Century Tunisian Sundial for Regulating the Times of Muslim Prayer), 18.

²⁰According to King in 'Aspects of Applied Science in Mosques and Monasteries,' (preprint) p. 14.

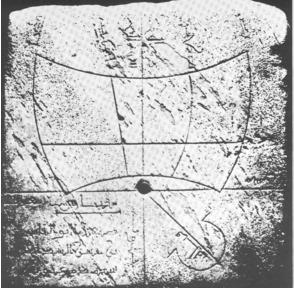
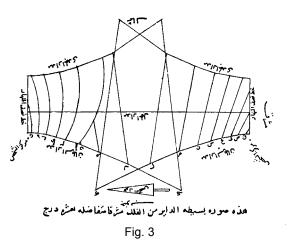


Fig. 2 courtesy of Francis Maddison (photo Alain Brieux)

Another example of a comprehensive solution to a set of dialing problems was the treatment of vertical dials for Cairo given by al-Marrakushi's contemporary, the astronomer Shihab al-Din al-Maqsi. This astronomer prepared a set of tables supplying, for each degree of declination to the local meridian for a vertical dial in Cairo, the coordinates of the points of intersection both of the lines for the seasonal hours and of the curve marking the time of the afternoon prayer with the shadow traces at the equinoxes and solstices.

Indeed, the late thirteenth century in Cairo seems to have been a time of considerable gnomonic activity, for in 1296 a beautiful variation on the common horizontal dial showing seasonal hours was installed in the ancient mosque of Ibn Tulun in that city.²¹ (Fig. 3 shows an illustration of a dial virtually identical to that of the Ibn Tulun mosque found in a manuscript copy of a treatise by the fifteenth-century timekeeper at a Cairo mosque, Ibn al-Muhallabi.) This dial, whose maker is unknown, has the net of lines and curves for such a dial split into two halves, with the midday lines for each half running along the

east and west sides of the plate and the net of hour lines spreading out toward the middle where they overlap. Not only are the line of the equinoxes and the hyperbolas of the solstices shown, but so are the hyperbolas indicating the sun's entry into each of the zodiacal signs.



At about the time the sundial of the Ibn Tulun mosque in Cairo was being constructed, an astronomer in the same city, Najm al-Din al-Misri, compiled a set of tables for timekeeping by the sun and stars in all localities, tables which one entered with three arguments and which contained more than 400,000 entries.²² With these tables he showed how to solve any problem of spherical astronomy involving a spherical triangle. These tables were also used to generate other tables for instrument construction by Najm al-Din in his treatise on astronomical instruments.²³

In the second half of the fourteenth century, the astronomer Ibn al-Shatir constructed for the great Umayyad Mosque in Damascus a horizontal sundial on a rectangular marble slate some 2 m x 1 m. (Fig. 4) This displayed time since sunrise in the morning, time before sunset in the afternoon, and time until or after midday. It was placed on one of the minarets of the mosque and completely oriented its user, who would have

²¹The dial, no longer extant, is described by L. Janin and D.A. King in *IAI*, Papere XVI ('Le Cadran Solaire de la Mosquée d'Ibn Tulun au Caire'). Plate 1 of that treatise shows a picture of the dial reassembled from fragments found in the wall of the Ibn Tulun mosque by scholars in Napoleon's expedition to Egypt. The fragments were subsequently stolen, and their whereabouts are unknown.

²² See François Charette, "A monumental medieval table for solving the problems of spherical astronomy for all latitudes," *Archives internationales d'histoire des sciences* 48, no. 140 (1998):11-64.

²³This treatise on instruments, including several for all latitudes, is being studied by François Charette for a Ph.D. thesis at The Johann Wolfgang Goethe University in Frankfurt a/M.

been the official timekeeper at the mosque, to time relative to the prayers during the day and at either end of it.

A surprising feature of Ibn al-Shatir's dial²⁴ was that its gnomon was parallel to the polar axis. In fact, it was inclined to the horizon at an angle of $33\frac{1}{2}^{\circ}$, a common medieval value for the latitude of Damascus. This was the first known proof that it was the Muslims, not the Europeans of the Renaissance, who introduced the sundial with the gnomon aligned parallel to the earth's axis. Like the earlier-mentioned dial of Ibn al-Saffar, the gnomon is modelled on the plane of the sundial, so it can be replaced if it is broken off.

Ibn al-Shatir's masterpiece actually incorporates three separate sundials. The first is a small one to the south (at the top in Fig. 4) whose western lines measure the number of hours since sunrise, whose eastern lines measure the number of hours until sunset, and whose gnomon is represented only by the southmost point of the main gnomon.

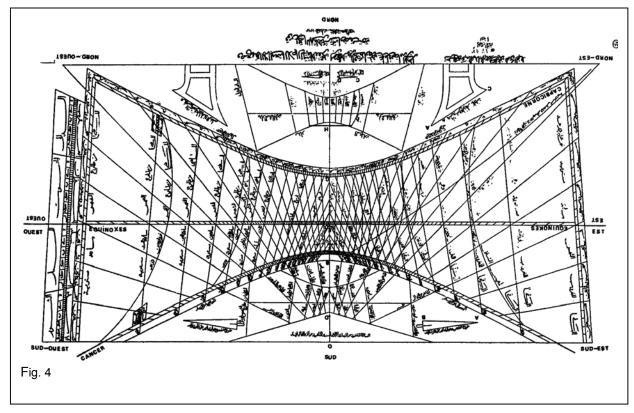
The second sundial on this instrument refines the

time given by the first. It is the central sundial, which again shows hours since dawn and hours until sunset and whose gnomon is represented by the northmost point of the main gnomon. The spaces between its hour lines are divided into intervals representing twenty minutes each.

The main gnomon serves both dials, and its sloping edge, rising from south to north, shows the equinoctial hours by its shadow falling on other lines marked on the central sundial.

North of the main gnomon was an auxiliary gnomon consisting of a post (mounted just north of the line of the winter solstice) with a southsloping blade at the top also inclined at the angle of the polar axis. Its role is simply to prolong to the line of the winter solstice the shadow of the main gnomon in order to measure smaller, fourminute, units of time.

The third dial is the northern one, served by a missing vertical gnomon. The shadow of the tip of the gnomon marks the seasonal hours in Damascus, and this dial too shows the curve for the afternoon prayers.



²⁴Based on the description in L. Janin, 'Le cadran solaire de la Mosquée Umayyade à Damas.' *Centaurus* 16 (1971): 285-98. Some of the remaining curves are clearly connected with prayer times. For example, of the seven curves on the east side of the dial, whose concavity faces the midday line, the furthest of these from that line marks the time of afternoon prayer. Another curve is labelled '13½ hours until dawn' and measures time relative to the prayer at daybreak.

The sundial now in the mosque in Damascus was made by an Egyptian astronomer who died in Damascus in 1889 and who built it after he had broken Ibn al-Shatir's original whilst trying to realign it. He compiled a new set of tables of coordinates for constructing such a sundial and was equal to the task of producing an exact replica of the original.

The French scholar Louis Janin wrote of Ibn al-Shatir's sundial that 'The drawing of its three dials, the richness of their lines and curves, the designs symmetrically set out of the gnomons and their models, the careful engraving of the numerous notes in perfectly calligraphed Arabic characters, the elegance of the gnomon, all of these confer on it a characteristically Muslim decorative appearance.' It was Ibn al-Shatir's crowning achievement, and four years after completing it he died.

A dial from Granada, although it is at the other end of the spectrum of quality from that of Ibn al-Shatir, and is rife with errors, contains among its several mistakes one particularly informative error, namely the use of circles rather than hyperbolas for its solsticial lines. D.A. Kina suggests, in defence of the designer, that he was working in a tradition, not otherwise known to us, in which, for dials showing only the few hours around noon needed for the prayers, one used circular arcs as reasonable approximations to hyperbolic arcs. If this is the case we have another example of cultural context influencing design. In any case, another example of such a dial is that of the dial in Tunis, illustrated in Fig. 2. Certainly drawing the full hyperbola was no easy task for an engraver and some simply used straight line segments in a continuous, roughly hyperbolic, path, as in an example from Medina Azahara, in al-Andalus, where the prayer lines are shown by zigzags to distinguish them from the hour lines²⁵ (Fig. 5).



Fig. 5 courtesy of Museo Arqueológico Provincial de Córdoba

One of the most ingenious anomonists in Jamshid medieval Islam was al-Kashi. astronomer royal for Ulug Beg in Samarkand. Al-Kashi writes in a letter to his father, around the vear 1400, somewhat scathingly of astronomers at the court who could only design vertical declining sundials by taking the time from other sundials and marking the appropriate hours on the desired surface.²⁶ To show his own virtuousity, he explains in a part of his Persian astronomical handbook, the Zij-i Khagani, devoted to determining the horoscope at a given time and locality, how to find the solar altitude from the width of shadow cast by a wall of arbitrary azimuth!²⁷ This virtuoso performance, designed to show off his skill, has nothing to do with utility.

the reader may consult King, *IAI*., and his articles 'Kibla' and 'Mizwala' in El_a.

²⁶E.S. Kennedy, 'A letter of Jamshid al-Kashi to His Father: Scientific Research and Personalities at a Fifteenth Century Court,' *Orientalia* 29 (1960): 195. (Reprinted in D.A. King and M. H. Kennedy (eds.), *Studies in the Islamic Exact Sciences: E.S. Kennedy, Colleagues and Former Students.* Beirut: American University of Beirut Press, 1983, 726.

²⁷ This, of course, specifies a unique time of day, provided one knows whether the measurement was taken in the forenoon or afternoon. Al-Kashi's method is explained in E.S. Kennedy and M.-Th. Debarnot, "Al-Kashi's Impractical Method of Determining the Solar Altitude," *Journal for the History of Arabic Science* 3 (1979): 219-27.

²⁵For further information on the Damascus dial, those of al-Andalus and the Maghrib, as well as many others,

The monumental astronomical instruments of Ulug Beg's observatory in Samarkand were the models for those that were later built in Jaipur, India, by the Mughal Emperor Jai Singh, whose monumental sundials, built in the 1730s, are undoubtedly the best known dials in the medieval Islamic tradition.²⁸

Finally, among the delights (and puzzles) of medieval gnomonics is a device known as 'the little ship' (navicula) in which one feeds in the date and the latitude and then needs only to hold the instrument up towards the sun and read off the time of day given correctly in equinoctial hours! The few surviving examples are of fourteenth-century English manufacture and found in European museums, but the modern scholar, who has investigated the device most thoroughly, David King, is confident that the device originated in Islamic lands. His argument is that our old friend, Habash, is known to have devised an instrument that served the same purpose for timekeeping by the stars. Habash's instrument is essentially a database of useful numerical information about various stars and a set of grids for reducing problems from three dimensions to one dimension in order to solve them.

Sundials, then, played a variety of roles in medieval Islam, not only as a source of intriguing problems for the geometer and a challenge to makers of tables for astronomical timekeeping, but also as a source of guidance for the mosque timekeeper and a familiar sight to worshippers at many mosques from Afghanistan to Morocco. This close connection to the daily practice of the Muslim faith gave sundials a place in Islamic culture quite different from that which they occupied in the earlier Greek and Roman civilizations, and the study of their history in medieval Islam combines scientific, religious, and other aspects of culture in a uniquely fascinating way.

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²⁸ These have been described and illustrated in H.E. Brandmaier, "Famous Sundials of India," *The Compendium* 2(4):4-9, Dec. 1995.