Problem #jh06.1

(This is part of **Math 252 Homework #06,** due Friday March 5th, 1999.)

Maxwell's equations in a vacuum (in the absence of any charges or currents) are given by:

$$\vec{\nabla} \bullet \vec{\mathbf{E}} = 0$$

$$\vec{\nabla} \bullet \vec{\mathbf{B}} = 0$$

$$\vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\vec{\nabla} \times \vec{\mathbf{B}} = \frac{1}{c^2} \frac{\partial \vec{\mathbf{E}}}{\partial t}$$

(a) Show that $\vec{\mathbf{E}}$ satisfies the wave equation:

$$\vec{\nabla}^2 \vec{\mathbf{E}} = \frac{1}{c^2} \frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2}$$

(b) Show that $\vec{\mathbf{B}}$ also satisfies the wave equation:

$$\vec{\nabla}^2 \vec{\mathbf{B}} = \frac{1}{c^2} \frac{\partial^2 \vec{\mathbf{B}}}{\partial t^2}$$

Assume:

$$\vec{\mathbf{E}} = \left[A\cos(\omega t - \vec{\mathbf{k}} \bullet \vec{\mathbf{R}}) + B\sin(\omega t - \vec{\mathbf{k}} \bullet \vec{\mathbf{R}}) \right] \hat{\mathbf{u}}_E$$

$$\vec{\mathbf{B}} = \left[C\cos(\omega t - \vec{\mathbf{k}} \bullet \vec{\mathbf{R}}) + D\sin(\omega t - \vec{\mathbf{k}} \bullet \vec{\mathbf{R}}) \right] \hat{\mathbf{u}}_B$$

where $\hat{\mathbf{u}}_E$ and $\hat{\mathbf{u}}_B$ are suitable unit vectors, $\vec{\mathbf{R}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$, ω is a constant, $\vec{\mathbf{k}}$ is a constant vector, and *A*, *B*, *C*, and *D* are constants.

(c) Show $\vec{\mathbf{k}} \perp \hat{\mathbf{u}}_E$

(d) Show $\vec{\mathbf{k}} \perp \hat{\mathbf{u}}_B$

(e) Show $\hat{\mathbf{u}}_E \perp \hat{\mathbf{u}}_B$

(f) Find a relation between *A* and *C*.

(g) Find a relation between *B* and *D*.

[Don't get $\vec{\mathbf{k}}$ confused with $\hat{\mathbf{k}}$. The former is the wave propagation vector, with a magnitude of $2\pi / \lambda$, whereas the latter is the unit vector parallel to the z axis.]