Math 252, Vector Calculus

J. Hebron, Spring 1999

Mid-Term Examination

Wednesday, Feb 17th, 1999

Marks

1. Prove, using tensor notation, the following vector identity: [10]

$$\vec{\nabla}\times(\vec{\mathbf{F}}\times\vec{\mathbf{G}})=(\vec{\mathbf{G}}\cdot\vec{\nabla})\vec{\mathbf{F}}-(\vec{\mathbf{F}}\cdot\vec{\nabla})\vec{\mathbf{G}}+(\vec{\nabla}\cdot\vec{\mathbf{G}})\vec{\mathbf{F}}-(\vec{\nabla}\cdot\vec{\mathbf{F}})\vec{\mathbf{G}}$$

2. Evaluate the following: [3]

$$\vec{\nabla}\times\vec{\nabla}\left(\frac{x^3y\sqrt{z}+e^{xy}\cos(yz)-\tanh^{-1}\left(\frac{x}{y}\right)}{\ln(x^2+z^2)+3xz^{4/3}y^{\pi\cos(x)}}\right)$$

3. What is $\vec{\nabla}\vec{\mathbf{F}}$ when written as a dyadic? [5]

4. Let $f(x,y,z)=\sin(px)\sinh(qy)e^{rz}$ where p, q, and r are constant. What are the necessary conditions on p, q, and r to make $f(x,y,z)$ satisfy Laplace's equation? [10]

5. Let $\vec{\mathbf{F}}=yz\vec{\mathbf{i}}+xz\vec{\mathbf{j}}+xy\vec{\mathbf{k}}$

   (a) What is $\vec{\nabla}\cdot\vec{\mathbf{F}}$? [2]

   (b) What is $\vec{\nabla}\times\vec{\mathbf{F}}$? [3]

   (c) What are the equations for the flow lines of $\vec{\mathbf{F}}$ going through the point (x_o, y_o, z_o) ? [5]

6. Find the equation of a plane tangent to the surface $z=x^2+y^2$ at the point (3,4,25). [10]

7. Consider the space curve defined by the following:

$$x=e^t\cos t$$
$$y=e^t\sin t$$
$$z=0$$

   and assume there is a particle moving along this curve as a function of time *t*.

   (a) What is the speed? [2]

   (b) What is the tangential component of acceleration? [2]

   (c) What is the normal component of acceleration? [2]

   (d) What is the unit tangent vector $\vec{\mathbf{T}}$? [2]

   (e) What is the curvature of the curve? [2]

   (f) What is the torsion of the curve? [2]

Total mark out of [60]