MATHEMATICS 151

Assignment 5, due Friday 09 July 1999

Section 2.3 (pg. 142):

12. $V = \frac{4}{3}\pi r^3$.

(a) The average rate of change of V with respect to r when r changes from 5 μm to 8 μm is $\dfrac{\frac{4}{3}\pi(8^3 - 5^3)}{8 - 5} = 172\pi\ \mu m^2 \approx 540.35\ \mu m^2$.

The average rate of change of V with respect to r when r changes from 5 μm to 6 μm is $\dfrac{\frac{4}{3}\pi(6^3 - 5^3)}{6 - 5} = \dfrac{364}{3}\pi\ \mu m^2 \approx 381.18\ \mu m^2$.

The average rate of change of V with respect to r when r changes from 5 μm to 5.1 μm is $\dfrac{\frac{4}{3}\pi(5.1^3 - 5^3)}{5.1 - 5} = \dfrac{7651}{75}\pi\ \mu m^2 \approx 320.48\ \mu m^2$.

(b) The instantaneous rate of change of V with respect to r when $r = 5$ μm is $4\pi r^2 = 100\pi \approx 314.16\ \mu m^2$.

16. $V = 5000\left(1 - \frac{t}{40}\right)^2$, $0 \le t \le 40$, with V measured in gallons and t in minutes.
Then $\dfrac{dV}{dt} = 10000\left(1 - \frac{t}{40}\right)\left(-\frac{1}{40}\right) = -250\left(1 - \frac{t}{40}\right)$.

(a) After 5 minutes the water is draining at a rate of 218.75 gal/min.

(b) After 10 minutes the water is draining at a rate of 187.5 gal/min.

(c) After 20 minutes the water is draining at a rate of 125 gal/min.

24. $n = 100 + 24t + 2t^2$, if n is the population measured in individual bacteria and t is the time measured in hours.
Then $\dfrac{dn}{dt} = 24 + 4t$, so when $t = 2$ h, the population growth rate is 32 bacteria/h.

30. If $C(x) = 2500 + 2\sqrt{x}$, $C'(x) = \dfrac{1}{\sqrt{x}}$, so $C'(100) = \dfrac{1}{10} = \$0.10$ per item.

$C(101) - C(100) = \left[2500 + 2\sqrt{101}\right] - \left[2500 + 2\sqrt{100}\right] = 2\left[\sqrt{101} - \sqrt{100}\right] = \dfrac{2}{\sqrt{101} + \sqrt{100}} \approx$
$\approx \$0.099751242$, slightly less than $C'(100)$. This is because $C'(x)$ continues to decrease between $x = 100$ and $x = 101$; most of the time while the 101st item is being made the marginal cost is less than what it is when the item is beginning to be made.

Section 2.4 (pg. 149):

2. $\lim_{x\to 0}(\cos(\sin x)) = 1$ since $\lim_{x\to 0}(\sin x) = \sin 0 = 0$ and $\lim_{x\to 0}(\cos x) = \cos 0 = 1$.

6. $\lim_{x\to 0}\frac{\sin x}{3x} = \frac{1}{3}\lim_{x\to 0}\frac{\sin x}{x} = \frac{1}{3}\cdot 1 = \frac{1}{3}$.

10. $\lim_{\theta\to 0}\frac{\cos\theta - 1}{\sin\theta} = \lim_{\theta\to 0}\frac{(\cos\theta - 1)(\cos\theta + 1)}{(\sin\theta)(\cos\theta + 1)} = \lim_{\theta\to 0}\frac{\cos^2\theta - 1}{(\sin\theta)(\cos\theta + 1)} =$

$= \lim_{\theta\to 0}\frac{-\sin^2\theta}{(\sin\theta)(\cos\theta + 1)} = \lim_{\theta\to 0}\frac{-\sin\theta}{\cos\theta + 1} = \frac{0}{1+1} = 0.$

28. If $y = 2x\left(\sqrt{x} - \cot x\right)$, $y' = 2\left(\sqrt{x} - \cot x\right) + 2x\left(\frac{1}{2}x^{-1/2} - (-\csc^2 x)\right) =$

$= 3\sqrt{x} - 2\cot x + 2x\csc^2 x.$

34. If $y = \sec x - 2\cos x$, $y' = (\sec x)(\tan x) + 2\sin x$, so $y'(\pi/3) = 2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$.

The tangent line has equation $y - 1 = 3\sqrt{3}\left(x - \frac{\pi}{3}\right)$, or $y = 3\sqrt{3}x + 1 - \sqrt{3}\pi$.

Section 2.5 (pg. 156):

2. $y = u^2 - 2u + 3$ and $u = 5 - 6x$.

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (2u - 2)(-6) = (8 - 12x)(-6) = -48 + 72x$, so $\left.\frac{dy}{dx}\right|_{x=1} = -48 + 72 = 24$.

Alternatively, $y = (5 - 6x)^2 - 2(5 - 6x) + 3 = 36x^2 - 48x + 18$, $y' = 72x - 48$,

and again $\left.\frac{dy}{dx}\right|_{x=1} = 24$.

8. $g(t) = (6t^2 + 5)^3(t^3 - 7)^4$.

$g'(t) = 3(6t^2 + 5)^2(12t)(t^3 - 7)^4 + (6t^2 + 5)^3 4(t^3 - 7)^3(3t^2) =$

$= 12(6t^2 + 5)^2(t^3 - 7)^3 t[3(t^3 - 7) + t(6t^2 + 5)] = 12(6t^2 + 5)^2(t^3 - 7)^3 t\,[9t^3 + 5t - 21].$

16. $s(t) = \sqrt[4]{\frac{t^3 + 1}{t^3 - 1}} = \left(\frac{t^3 + 1}{t^3 - 1}\right)^{1/4}$.

$s'(t) = \frac{1}{4}\left(\frac{t^3 + 1}{t^3 - 1}\right)^{-3/4}\cdot\frac{(t^3 - 1)(3t^2) - (t^3 + 1)(3t^2)}{(t^3 - 1)^2} = \frac{1}{4}\left(\frac{t^3 + 1}{t^3 - 1}\right)^{-3/4}\cdot\frac{-6t^2}{(t^3 - 1)^2} = -\frac{3}{2}\frac{t^2}{(t^3 + 1)^{3/4}(t^3 - 1)^{5/4}}.$

26. If $y = \tan(x^2) + \tan^2 x$, $y' = \sec^2(x^2)\cdot(2x) + (2\tan x)(\sec^2 x)$.

42. If $y = \sin(\sin(\sin x))$, $y' = \cos(\sin(\sin x))\cdot\cos(\sin x)\cdot(\cos x)$.

62. $s = A\cos(\omega t + \delta)$.

(a) $v = s' = -\omega A\sin(\omega t + \delta)$.

(b) $v = 0$ when $\sin(\omega t + \delta) = 0$.

This is when $t = \frac{n\pi - \delta}{\omega}$, n any integer, assuming $A \neq 0$ and $\omega \neq 0$.

68. If $g(t) = [f(\sin t)]^2$, $g'(t) = 2f(\sin t)\cdot f'(\sin t)\cdot(\cos t)$.