

Math 251, Calculus III

J. Hebron, Fall 1999

Mid-Term Examination #2

Wednesday, Nov 3rd, 1999

Time: 50 minutes

Student ID Number

Name

(Please underline your family name)

Signature

Instructions:

- **Please fill-in the above information in ink.**
- **Do not open this exam until told to do so.**
- **No books, no notes, no calculators**
- **Please sign the bottom of every page**
(in case your exam becomes unstapled)

Question #:	1	2	3	4	5	6	7	Tot
Mark:								
Out of:	6	6	6	8	8	8	8	50

1. Let $f(x, y) = \frac{2x^2 - 3xy + y^2}{x^2 + y^2}$. Show $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ is path-dependent.

Find two paths through $(0,0)$ along which this limit is 0.

[6]

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2. Let $f(x, y) = \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + a^2} - a}$, $a > 0$. Find $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$. Verify that the limit is unique, using the ϵ - δ method.

[6]

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3. Let $z(x,y)$ be defined implicitly by $y^2ze^{(x+y)} - \sin(xyz) = 0$. Find $\frac{z}{x}$.

[6]

(Signature)

4. The shape of a volcano is given by $z = he^{-(x^2 + 2y^2)}$, where $h > 0$ is its maximum height. The volcano erupts and lava is flowing. In what direction, in the xy -plane, is the lava flowing at $x=1, y=2$? Express the direction as a unit vector in the xy -plane.

[8]

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5. Let $f(x, y) = 2\sqrt{x^2 + |y|} - x^2$. Find all the critical points.

[8]

(Signature)

6. Let $f(x, y) = x^2 + 3y^4 - 4y^3 - 12y^2$. Find all critical points and classify as to whether they correspond to local maxima, local minima, or saddle points.

[8]

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7. Use the method of Lagrange Multipliers to find the dimensions of the right rectangular box of largest volume which can be inscribed inside a sphere of radius r .

[8]

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