Math 314, Boundary Value Problems

J. Hebron, Fall 1999

Mid-Term Examination

Wednesday, Oct 20th, 1999

This is a take-home exam. It is due back at the start of class on Wednesday, Oct 27th, 1999. You are allowed to consult your books. However, please work on your own and do not discuss this with your classmates. It is to be treated as an exam. Letter grades will be assigned on a relative basis.

Please show all your work. Submit your answers in a duo-tang or similar type of folder, with numbered pages. Please leave a mostly blank page at the front with your name on it, for recording the marks.

Marks

1. Chapter 0, Miscellaneous Exercise #23 (b), page 53. Show that the answer given in the back of the book is wrong. Can you find the correct answer? [5]

2. Chapter 0, Miscellaneous Exercise #24, page 53. Let $f(x) = -\gamma^2 x$ and compare the solution obtained by this method to the solution obtained by direct methods. [5]

3. Explain what you understand of the concept of orthogonal functions. [10]

4. Suppose $\varphi_n(x) = \sin(\lambda_n x)$ where $\lambda_n = \dfrac{(n-\alpha)\pi}{a}$, $n = 1,2,3,\ldots$, $0 \le \alpha < 1$. If you want $\varphi_n(x)$ to be an orthogonal set of functions on $(-a,a)$, what possible values can α have? [10]

5. Devise an infinite set of orthogonal functions on (0,a), composed entirely of straight-line segments. Normalize them. Try to expand $f(x) = \sin\left(\dfrac{\pi x}{a}\right)$ in terms of your orthonormal set. Work out the first 4 generalized Fourier coefficients. [20]

6. Chapter 1, Miscellaneous Exercise #4, page 124. [5]

7. Chapter 1, Miscellaneous Exercise #24, page 127. [5]

8. Chapter 2, Miscellaneous Exercise #2, page 208. [10]

9. Chapter 2, Miscellaneous Exercise #4, page 209. [10]

10. Chapter 2, Miscellaneous Exercise #10, page 210. [10]

11. Chapter 2, Miscellaneous Exercise #16, page 211. [10]

Total mark out of [100]