Definition of a vector space over $\mathbb{R}$

A *vector space over* $\mathbb{R}$ is a set V of vectors together with a distinguished vector $\mathbf{0}$ in V and three functions

$$(\boldsymbol{u}, \boldsymbol{v}) \mapsto \boldsymbol{u} + \boldsymbol{v} \qquad (\boldsymbol{u},\, \boldsymbol{v} \in V)$$

$$\boldsymbol{v} \mapsto -\boldsymbol{v} \qquad (\boldsymbol{v} \in V)$$

$$(r, \boldsymbol{v}) \mapsto r\boldsymbol{v} \qquad (r \in \mathbb{R}, \boldsymbol{v} \in V)$$

which satisfy

A1	$(\boldsymbol{u} + \boldsymbol{v}) + \boldsymbol{w} = \boldsymbol{u} + (\boldsymbol{v} + \boldsymbol{w})$	associative law
A2	$\boldsymbol{u} + \boldsymbol{v} = \boldsymbol{v} + \boldsymbol{u}$	commutative law
A3	$\mathbf{0} + \boldsymbol{u} = \boldsymbol{u}$	additive identity
A4	$\boldsymbol{u} + (-\boldsymbol{u}) = \mathbf{0}$	additive inverse
S1	$r(\boldsymbol{u} + \boldsymbol{v}) = r\boldsymbol{u} + r\boldsymbol{v}$	distributivity
S2	$(r + s)\boldsymbol{u} = r\boldsymbol{u} + s\boldsymbol{u}$	distributivity
S3	$r(s\boldsymbol{u}) = (rs)\boldsymbol{u}$	associative law
S4	$1\boldsymbol{u} = \boldsymbol{u}$	scale preservation

for all $\boldsymbol{u}$, $\boldsymbol{v}$, $\boldsymbol{w} \in V$ and r, $s \in \mathbb{R}$.