SIMON FRASER UNIVERSITY

DEPARTMENT OF MATHEMATICS AND STATISTICS

First Midterm

MATH 232

June 3, 1998, 11:30 − 12:20 a.m.

Name: _____ (please print)

family name   *given name*

Signature: _____

INSTRUCTIONS

1. Write your name above in block letters and sign below your name.

2. Record your answers on the answer pages found immediately below this cover sheet.

3. No calculators or other computing devices may be used.

4. This exam has 9 questions on 7 pages which follow the answer pages — please check to make sure your exam is complete.

5. Ask for clarification if you cannot understand the question or there appears to be an error.

6. If the space provided for rough work is insufficient you may use the back of the previous page.

ANSWER PAGE 1

QUESTION	Answer	MAX	SCORE
1		3	
2		4	
3		4	
4	(a) (b)	2 2	
5		5	
6		5	

SUBTOTAL	

Question	Answer	Max	Score
7		5	
8		5	
9	Yes, the set is a basis of its span ☐ No, it is not ☐ Brief reason:	5	

Subtotal	

Exam total	

[3] **1. Compute the vector**
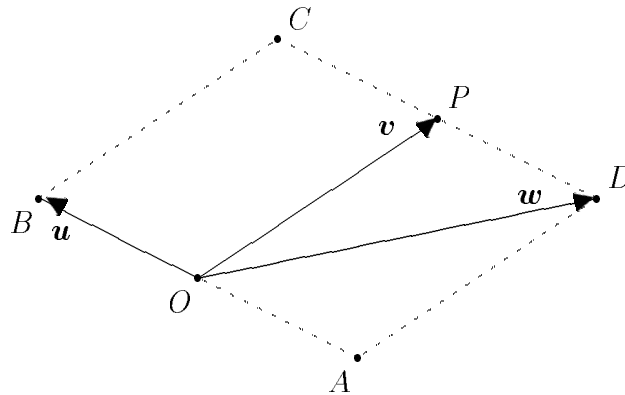
$$3[1, -3, 4] - 4[-1, -1, 2] + [3, 3, 3].$$

(The answer should be a vector in $\mathbb{R}^3$.)

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[4] **2.** The diagram below show some vectors which lie in the same plane in $\mathbb{R}^3$. $ABCD$ is a parallelogram. O, the origin, which represents the zero vector, is the midpoint of AB, and P is the midpoint of CD.

$\boldsymbol{u}$, $\boldsymbol{v}$, $\boldsymbol{w}$ are the vectors represented by the points B, P, D respectively.

Write $\boldsymbol{w}$ as a linear combination of $\boldsymbol{u}$ and $\boldsymbol{v}$.

(Your answer should have the form: $\boldsymbol{w} = ?\boldsymbol{u} + ?\boldsymbol{v}$.)

Enter your answer on the answer sheets following the cover page

[4]   **3. Write the following linear system as a column vector equation:**

$$\left. \begin{array}{rrrrrrr} 3x_1 & - & x_2 & + & 4x_3 & = & 9 \\ & & 2x_2 & - & 5x_3 & = & -12 \\ 4x_1 & - & 2x_2 & + & x_3 & = & 10 \end{array} \right\}$$

ROUGH WORK

Enter your answer on the answer sheets following the cover page

4. Consider the vectors

$$\boldsymbol{u} = [5, -3, -1, 1], \ \boldsymbol{v} = [2, 2, 4, 4], \ \boldsymbol{w} = [3, 3, 1, 1], \ \boldsymbol{x} = [1, 1, 1, -1].$$

[2]   (a) **Find one of these vectors which has length** 6.

[2]   (b) **Find two of these vectors which are orthogonal.**

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[5]   **5. Find the reduced row-echelon form of the matrix**

$$\begin{bmatrix} 2 & 1 & 3 & -1 \\ -1 & 2 & 2 & -2 \\ 5 & 0 & 4 & 0 \end{bmatrix}.$$

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[5] **6.** Consider the linear system $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$,

where $A \in \mathbb{R}^{3 \times 4}$ and $b_1, b_2, b_3 \in \mathbb{R}$.

The reduced row-echelon form of the augmented matrix of the system is

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & -1 & 2 \\ 0 & 0 & 1 & 5 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right].$$

Write down the general solution of the system.

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[5] **7.** Let A denote the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

Express A as a product of elementary matrices.

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[5]   **8. Find a basis for the solution set of the linear system**

$$\left. \begin{array}{ccccccccc} x_1 & - & 2x_2 & + & x_3 & - & x_4 & = & 0 \\ 2x_1 & - & 3x_2 & + & 4x_3 & - & 3x_4 & = & 0 \\ 3x_1 & - & 5x_2 & + & 5x_3 & - & 4x_4 & = & 0 \, . \end{array} \right\}$$

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[5]   **9. Determine whether the set of vectors $\{v_1, v_2, v_3\}$ is a basis for the subspace of $\mathbb{R}^4$ spanned by this set, where**

$$v_1 = [2, -1, 3, -4], \ \ v_2 = [3, -2, 5, -7], \ \ v_3 = [-1, 2, -3, 5].$$

Give a brief reason for your answer.

ROUGH WORK

Enter your answer on the answer sheets following the cover page