SIMON FRASER UNIVERSITY

DEPARTMENT OF MATHEMATICS AND STATISTICS

Sample First Midterm

MATH 232

May, 1998, 55 minutes

Name: _____ (please print)

family name *given name*

Signature: _____

INSTRUCTIONS

1. Write your name above in block letters and sign below your name.
2. Record your answers on the two answer sheets attached to this cover sheet.
3. No calculators or other computing devices may be used.
4. This exam has 9 questions on 2 pages which are separate from the answer pages — please check to make sure your exam is complete.
5. Ask for clarification if you cannot understand the question or there appears to be an error.
6. Two blank sheets are appended to the question paper which may be used for your rough work.

Answer Page

Question	**Answer**	Max	Score
1		3	
2		4	
3		4	
4		4	
5		5	
6	Yes, $\boldsymbol{b}$ is in the span ☐ No, $\boldsymbol{b}$ is not in the span ☐ Brief reason:	5	

Subtotal	

Answer Page 2

Question	Answer	Max	Score
7		5	
8		5	
9		5	

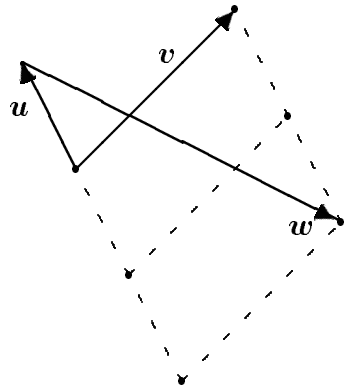
Subtotal	

Exam total	

[3] **1. Find a real number c such that**

$$c\,[2,-2,4] + [3,c,5] = [1,1,1]\,.$$

[4] **2.** In the vector diagram below the lines which look parallel are parallel, and the line segments which look equal are equal.



Write w as a linear combination of u and v.

[4] **3. Compute the matrix product AB when A, B are the $n \times n$ matrices given by**

$$A = \begin{bmatrix} 1 & a_{12} & a_{13} & \dots & a_{1n} \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & b_{12} & b_{13} & \dots & b_{1n} \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}.$$

[5] **4. Find the reduced row-echelon form of the matrix**

$$\begin{bmatrix} 0 & -1 & 2 & 2 \\ 1 & 0 & 2 & 0 \\ 2 & 2 & 0 & -4 \end{bmatrix};$$

[5] **5. For what value(s) of c is the following system consistent?**

$$\left.\begin{array}{rcrl} 3x & + & y & = 1 \\ x & - & 2y & = c \\ 2x & + & y & = 3 \end{array}\right\}$$

[4] **6. Determine whether b is in the span of the vectors v_1, v_2, v_3, where**

$$\boldsymbol{b} = \begin{bmatrix} 3 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \ \boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix}, \ \boldsymbol{v}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \ \boldsymbol{v}_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \\ -2 \end{bmatrix}.$$

Give a brief reason for your answer.

[5] **7.** Consider the linear system

$$A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, \tag{1}$$

where $A \in \mathbb{R}^{3\times 4}$ and $b_1, b_2, b_3 \in \mathbb{R}$.

The reduced row-echelon form of the augmented matrix of the system is

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & -2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

Write down the general solution of the system (1).

[5] **8.** Let A denote the matrix $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$.

Express A^{-1} as a product of elementary matrices.

[5] **9. Find a basis for the solution set of the linear system**

$$\left.\begin{array}{rcrcrcrcl} x_1 & + & x_2 & + & x_3 & - & x_4 & = & 0 \\ & - & x_2 & + & 4x_3 & - & 3x_4 & = & 0 \\ 2x_1 & + & x_2 & + & 6x_3 & - & 5x_4 & = & 0\,. \end{array}\right\}$$

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