SIMON FRASER UNIVERSITY

DEPARTMENT OF MATHEMATICS AND STATISTICS

First Midterm

MATH 232

February 3, 1999, 11:30 – 12:20 a.m.

Name: ______________________________ (please print)

family name *given name*

Signature: ______________________________

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Write your name above in block letters and sign below your name.
   Write your student number in the box on the inside of the back cover page.
3. For each question write your final answer in the box provided.
4. No calculators or other computing devices may be used.
5. This exam has 10 questions on 7 pages — please check to make sure your exam is complete.
6. If the space provided for rough work is insufficient you may use the back of the previous page.

[3] **1. Compute a vector $\boldsymbol{x} \in \mathbb{R}^3$ such that**

$$[1, -2, 1] - 2\boldsymbol{x} = [5, 2, -3].$$

ANSWER

ROUGH WORK

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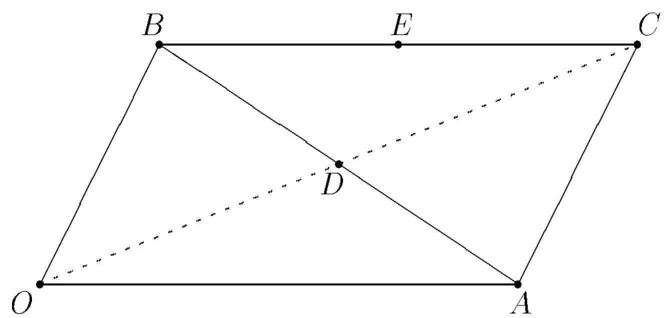
2. The diagram below shows points O, A, B, C, D, E in $\mathbb{R}^3$, O being the origin. The figure $OACB$ is a parallelogram, D is the midpoint of the line segment AB, and E is the midpoint of BC.

The points A, B have coordinate vectors $\boldsymbol{a}$, $\boldsymbol{b}$, respectively, while O has coordinate vector $\mathbf{0} = [0, 0]$.



[2] (a) **Write down the coordinate vector of D in terms of $\boldsymbol{a}$, $\boldsymbol{b}$.**

ANSWER

[2] (b) **Write down the coordinate vector of E in terms of $\boldsymbol{a}$, $\boldsymbol{b}$.**

ANSWER

ROUGH WORK

[2] **3.** (a) **Compute the dot product**

$$[4, 1, -1, 2] \cdot [-1, 3, -3, -1].$$

ANSWER

[2] (b) **Compute the norm**

$$\|[-1, 7, 5, 6, -1, 3]\|.$$

ANSWER

[2] (c) **Find a unit vector orthogonal to**

$$[1, -2, 1].$$

ANSWER

ROUGH WORK

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[2] **4.** (a) **Write down the augmented matrix of the system:**

$$\begin{cases} 2x_1 + x_2 + x_3 + x_4 = 2 \\ 4x_1 + 2x_2 \phantom{{}+x_3} + 2x_4 = 4 \end{cases}$$

ANSWER

[2] (b) **Convert the matrix from (a) to reduced row-echelon form by row operations.**

Write the final answer in the answer box.

ANSWER

ROUGH WORK

5. Consider the system of linear equations $A\boldsymbol{x} = \boldsymbol{b}$, where $\boldsymbol{x} = [x_1, x_2, x_3, x_4, x_5]$, $\boldsymbol{b} \in \mathbb{R}^5$, and $A \in \mathbb{R}^{4\times 5}$.

It is given that the reduced row-echelon form of the augmented matrix $[A|\boldsymbol{b}]$ is the matrix.

$$\left[\begin{array}{ccccc|c} 1 & 2 & 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

[5] (a) **Find the general solution of the system $A\boldsymbol{x} = \boldsymbol{b}$ writing your final answer in vectorial form in the box below.**

ANSWER

$\boldsymbol{x} =$

[2] (b) **Using your answer to (a), write down the general solution of the homogeneous system $A\boldsymbol{x} = 0$ writing your final answer in vectorial form in the box below.**

ANSWER

$\boldsymbol{x} =$

ROUGH WORK

(continue on back of page 2 if you need more room)

6. Let A denote the matrix:

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

[3] (a) **Find row operations ρ_1, ρ_2, ..., ρ_k such that $\rho_k(\ldots\rho_2(\rho_1(A))\ldots) = I$.**

This can be done with three row operations but one might use more.

ANSWER

$\rho_1 =$

$\rho_2 =$

$\rho_3 =$

$\rho_4 =$

$\rho_5 =$

$\rho_6 =$

ROUGH WORK

[2] (b) **Based on your answer to (a), express A as a product of elementary matrices.**

ANSWER

$A \;=$

[2] (c) **Based on your answer to (b), express A^{-1} as a product of elementary matrices.**

ANSWER

$A^{-1} \;=$

[3] **7.** Let vectors $\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_k$ in $\mathbb{R}^n$ be given.

Describe a procedure for finding a basis of $\mathrm{sp}(\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_k)$.

ANSWER

[2] **8.** Let A be the 4×5 matrix from question 5 on page 3.

Write down a basis for the nullspace of A which is defined to be

$$\{\boldsymbol{x} \in \mathbb{R}^5 \ : \ A\boldsymbol{x} = \boldsymbol{0}\}$$

ANSWER

[2] **9.** The definition of *basis* says that $\{\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_k\}$ is a basis for the subspace V of $\mathbb{R}^n$ if $\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_k \in V$ are distinct and for every $\boldsymbol{v} \in V$ there are unique $c_1, c_2, \ldots, c_k$ in $\mathbb{R}$ such that

$$c_1\boldsymbol{a}_1 + \ldots + c_k\boldsymbol{a}_k = \boldsymbol{v}\,.$$

Show from first principles that $\mathbb{R}^5$ has a basis of size 5.

ANSWER

[2] **10.** In this question you may assume the conclusion of question 9 and that *every* basis of $\mathbb{R}^5$ has size 5. You may **not** assume anything else.

Let $\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3, \boldsymbol{b}_4$ be distinct linearly independent vectors in $\mathbb{R}^5$.

Prove that there exists a vector $\boldsymbol{c}$ in $\mathbb{R}^5$ such that $\{\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3, \boldsymbol{b}_4, \boldsymbol{c}\}$ is linearly independent.

ANSWER

Student number

Family name

DO NOT WRITE BELOW THIS LINE

Question	Maximum	Score
1	3	
2	4	
3	6	
4	4	
5	7	
6	7	
7	3	
8	2	
9	2	
10	2	
Total	40	