SIMON FRASER UNIVERSITY

DEPARTMENT OF MATHEMATICS AND STATISTICS

First Midterm

MATH 232

October 6, 1999, 11:30 – 12:20 a.m.

Name: _____ (please print)

family name *given name*

Signature: _____

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Write your name above in block letters and sign below your name. Write your student number and your family name in the boxes on the inside of the back cover page.
3. For each question write your final answer in the box provided.
4. No calculators or other computing devices may be used.
5. This exam has 9 questions on 7 pages — please check to make sure your exam is complete.
6. If the space provided for rough work is insufficient you may use the back of the previous page.

[2] **1. Compute a vector $\boldsymbol{x} \in \mathbb{R}^3$ such that**

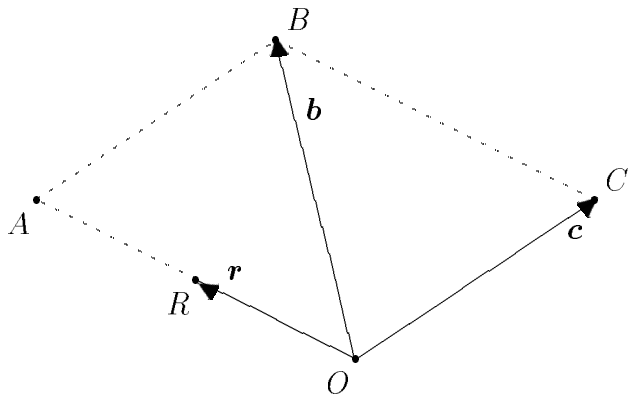
$$3\boldsymbol{x} - [2, 1, -1] = [7, -2, 4].$$

ANSWER

ROUGH WORK

[4] **2. The diagram below shows some vectors which lie in the same plane in $\mathbb{R}^3$. $OABC$ is a parallelogram. The origin O represents the zero vector. R is the midpoint of OA.**

$\boldsymbol{r}$, $\boldsymbol{b}$, $\boldsymbol{c}$ are the vectors represented by the points R, B, C respectively.



Write c as a linear combination of r and b.
Your answer should have the form: $c = ?r + ?b$.
(Hint: You can start by expressing b as a linear combination of r and c.)

ANSWER

$\boldsymbol{c} =$

ROUGH WORK

[2] **3.** (a) **Find all values $c \in \mathbb{R}$ such that the vectors $[1, -4, 7]$ and $[-3, c, 5]$ are orthogonal.**

ANSWER

[2] (b) **Compute the norm**

$$\|[-3, 2, 4, 0, -2]\|.$$

ANSWER

[2] (c) **Find a unit vector parallel to the vector**

$$[-2, 1, 2, -4].$$

ANSWER

ROUGH WORK

---

[2] **4.** (a) **Write down the augmented matrix of the system:**

$$\begin{cases} 2x_1 - 2x_2 - 4x_3 = -6 \\ x_1 + 2x_2 + 4x_3 = 3 \\ 4x_2 + 11x_3 = 8 \end{cases}$$

ANSWER

[3] (b) **Convert the matrix from (a) to reduced row-echelon form by row operations.**

Write the final answer in the answer box.

ANSWER

ROUGH WORK - *use back of page 1*

[3] **5. Consider the system of linear equations $A\boldsymbol{x} = \boldsymbol{b}$, where $\boldsymbol{x} = [x_1, x_2, x_3, x_4]^T$, $\boldsymbol{b} \in \mathbb{R}^2$, and $A \in \mathbb{R}^{2\times 4}$.**

It is given that the reduced row-echelon form of the augmented matrix $[A|\boldsymbol{b}]$ is the matrix

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & -3 & 9 \\ 0 & 0 & 1 & 7 & -4 \end{array}\right]$$

Find the general solution of the system $A\boldsymbol{x} = \boldsymbol{b}$ writing your final answer in vectorial form in the box below.

ANSWER

$\boldsymbol{x} =$

ROUGH WORK

6. Let A denote the matrix:

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$$

[4] (a) **Compute A^{-1}, the inverse matrix of A.**
(Give your answer in the form of a single 3×3 matrix.)

ANSWER

$A^{-1} =$

ROUGH WORK

[2] (b) **Based on the elementary row operations that you used to answer the part (a), express the matrix A as a product of elementary matrices.**

ANSWER

$A =$

[2] **7.** (a) **Let** $V = \{[3x - y, -x + y, 2x] \ : \ x, y \in \mathbb{R}\}$
be the set of all vectors $[3x - y, -x + y, 2x]$ **where** x, y **are arbitrary real numbers.**
Decide whether V **is a subspace of** $\mathbb{R}^3$**. Give a reason for your answer.**

ANSWER

[2] (b) **Let** $W = \{[-2x + y, y, 1, x - y] \ : \ x, y \in \mathbb{R}\}$
be the set of all vectors $[-2x + y, y, 1, x - y]$ **where** x, y **are arbitrary real numbers.**
Decide whether W **is a subspace of** $\mathbb{R}^4$**. Give a reason for your answer.**

ANSWER

[5] **8. Given is the following system of equations:**

$$\left.\begin{array}{rcrcrcrcl} x_1 & + & 2x_2 & - & 4x_3 & - & 3x_4 & = & 0 \\ 3x_1 & + & 7x_2 & - & 15x_3 & - & 8x_4 & = & 0 \end{array}\right\}$$

Let W **be the set of all solutions (regarded as column vectors) to this linear system. Find a basis for** W**.**

ANSWER

ROUGH WORK - *use back of page 5*

[5] **9. Determine whether the set of vectors $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ is a basis for the subspace of $\mathbb{R}^4$ spanned by this set, where**

$$\boldsymbol{v}_1 = [2, 1, -3, 4],\ \boldsymbol{v}_2 = [1, -1, 0, 5],\ \boldsymbol{v}_3 = [1, 5, -6, -7].$$

Give a reason for your answer.

ANSWER

ROUGH WORK

Student number

Family name

DO NOT WRITE BELOW THIS LINE

Question	Maximum	Score
1	2	
2	4	
3	6	
4	5	
5	3	
6	6	
7	4	
8	5	
9	5	
Total	40	