Math 232, Elementary Linear Algebra

J. Hebron, Spring 2000

Mid-Term Examination #1

Wednesday, February 2nd, 2000

Time: 50 minutes

	SOLUTIONS
Student ID Number	Name (Please underline your family name)
	J.H.
	Signature

Instructions:

- **Please fill-in the above information in ink.**
- **Do not open this exam until told to do so.**
- **No books, no notes, no calculators**
- **Please sign the bottom of every page**
  (in case your exam becomes unstapled)

Question #:	1	2	3	4	5	6	7	8	Tot
Mark:									
Out of:	10	9	4	10	8	7	7	20	75

Part A: Multiple Choice

Instructions: Circle the correct answer for each question. You may use the back pages of the exam for any rough work. Note that rough work will not be marked. Marks are only awarded for the correct answer.

[mark]

1. Let $\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$, and $\vec{\mathbf{w}}$ be vectors in $\mathbb{R}^3$ given by:

$$\vec{\mathbf{u}} = [0, 4, -3] \qquad \vec{\mathbf{v}} = [-3, 0, 4] \qquad \vec{\mathbf{w}} = [1, 1, x]$$

(i) What is the value of x such that $\vec{\mathbf{w}}$ is in the span of $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$? [4]

(a) 0 **(b)** -3 **(c)** 4 **(d)** -3/4 **(e)** -4/3 **(f)** 3/4 **(g)** 4/3

(h) 3/4 + 4/3 **(i) -3/4 - 4/3**

(j) not in the span for any value of x

(k) none of the above

(ii) What is the value of x such that $\vec{\mathbf{w}}$ is orthogonal to $\vec{\mathbf{u}}$? [2]

(a) 0 **(b)** -3 **(c)** 4 **(d)** -3/4 **(e)** -4/3 **(f)** 3/4 **(g) 4/3**

(h) 3/4 + 4/3 **(i)** -3/4 - 4/3

(j) not orthogonal for any value of x

(k) none of the above

(iii) What is the value of x such that $\vec{\mathbf{w}}$ is orthogonal to $\vec{\mathbf{v}}$? [2]

(a) 0 **(b)** -3 **(c)** 4 **(d)** -3/4 **(e)** -4/3 **(f) 3/4** **(g)** 4/3

(h) 3/4 + 4/3 **(i)** -3/4 - 4/3

(j) not orthogonal for any value of x

(k) none of the above

(iv) What is the value of x such that $\vec{\mathbf{w}}$ is parallel to $\vec{\mathbf{u}}$? [1]

(a) 0 **(b)** -3 **(c)** 4 **(d)** -3/4 **(e)** -4/3 **(f)** 3/4 **(g)** 4/3

(h) 3/4 + 4/3 **(i)** -3/4 - 4/3

(j) not parallel for any value of x

(k) none of the above

(v) What is the value of x such that $\vec{\mathbf{w}}$ is parallel to $\vec{\mathbf{v}}$? [1]

(a) 0 **(b)** -3 **(c)** 4 **(d)** -3/4 **(e)** -4/3 **(f)** 3/4 **(g)** 4/3

(h) 3/4 + 4/3 **(i)** -3/4 - 4/3

(j) not parallel for any value of x

(k) none of the above

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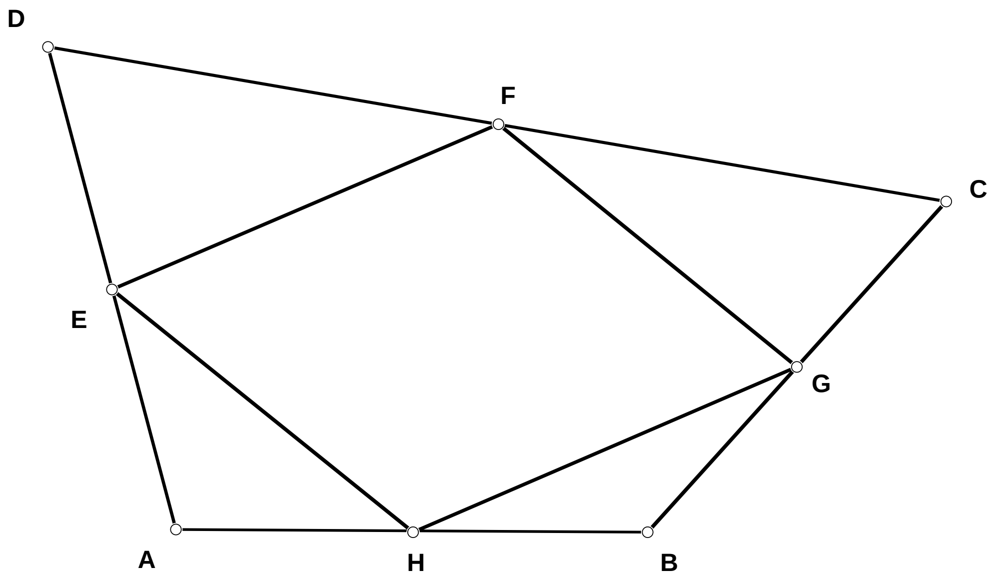
2. In the following diagram, **H** is the midpoint of segment **AB**, **G** is the midpoint of segment **BC**, **F** is the midpoint of segment **CD**, and **E** is the midpoint of segment **DA**.

The segments in this diagram will be interpreted as vectors. Eg. $\vec{AB}$ is the vector from **A** to **B**, whereas $\vec{BA}$ is the vector from **B** to **A**.



Indicate whether the following statements are true, false, or if it can't be said either way due to a lack of information.

(i)	$\vec{AB}+\vec{BC}+\vec{CD}+\vec{DA}=\vec{0}$	**(a) true**	**(b)** false	**(c)** can't say	[1]
(ii)	$\vec{DA}+\vec{DC}+\vec{AB}+\vec{CB}=\vec{0}$	**(a)** true	**(b) false**	**(c)** can't say	[1]
(iii)	$\vec{EF}=\vec{ED}+\vec{DF}$	**(a) true**	**(b)** false	**(c)** can't say	[1]
(iv)	$\vec{EF}=\vec{DF}-\vec{ED}$	**(a)** true	**(b) false**	**(c)** can't say	[1]
(v)	$\vec{EF}=\vec{AF}-\vec{EA}$	**(a)** true	**(b) false**	**(c)** can't say	[1]
(vi)	$\vec{DF}=\vec{FC}$	**(a) true**	**(b)** false	**(c)** can't say	[1]
(vii)	$\vec{EF}=\vec{GH}$	**(a)** true	**(b) false**	**(c)** can't say	[1]
(viii)	$\vec{FE}\bullet\vec{GF}=\vec{HE}\bullet\vec{HG}$	**(a)** true	**(b) false**	**(c)** can't say	[1]
(ix)	$\vec{EF}\bullet\vec{EH}=\vec{GF}\bullet\vec{GH}$	**(a) true**	**(b)** false	**(c)** can't say	[1]

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3. Indicate whether the following statements are true or false.

(i) The span of any two nonzero, nonparallel vectors in $\mathbb{R}^2$ is all of $\mathbb{R}^2$. [2]

(a) true **(b)** false

(ii) The span of any three nonzero, nonparallel vectors in $\mathbb{R}^3$ is all of $\mathbb{R}^3$. [2]

(a) true **(b) false**

4. Let $\mathbf{A}$ and $\mathbf{B}$ be the following matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

For what value of r does $\mathbf{A}$ commute with $\mathbf{B}$? [10]

(a) 0 **(b) 1** **(c)** -1 **(d)** 5 **(e)** 1/5 **(f)** 1 and 5 **(g)** no values

(h) all real numbers **(i)** none of the above

5. Suppose you are given a linear system of 4 equations in 4 unknowns x_1, x_2, x_3, and x_4. Further, suppose you have formed the augmented matrix, and have done Gauss Reduction to obtain the following row-echelon matrix:

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

What is the value of x_1 ? [8]

(a) -5 **(b)** -4 **(c)** -3 **(d)** -2 **(e)** -1 **(f) 0** **(g)** 1 **(h)** 2 **(i)** 3

(j) 4 **(k)** 5 **(l)** inconsistent **(m)** free **(n)** none of the above

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6. Let **A** be the following 4 by 3 matrix:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Which of the following forms a basis for the nullspace of **A** ? [7]

(a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix} \right\}$ **(b)** $\{[1,2,3],[0,1,2]\}$ **(c)** $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \\ 0 \end{bmatrix} \right\}$

(d) $\{[1,0,-1],[0,1,2]\}$ **(e)** $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}$ **(f)** $\left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \right\}$

(g) $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ **(h)** $\left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ **(i)** none of the above

7. Let **B** be the following 3 by 4 matrix: [7]

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Which of the following forms a basis for the nullspace of **B** ?

(a) $\left\{ \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$ **(b)** $\left\{ \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}$ **(c)** $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} \right\}$ **(e)** $\left\{ \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} \right\}$ **(f)** $\left\{ \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} \right\}$

(g) $\{[1,0,2,3],[0,1,-3,-2]\}$ **(h)** $\{[0,0,1]\}$ **(i)** none of the above

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Part B: Show All Your Work

Instructions: Work out the following problem, showing all your work. Part marks will be awarded even if the final answer is wrong.

8. Let **C** be the following 3 by 3 matrix:

$$\mathbf{C} = \begin{bmatrix} -1 & 1 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

(a) Find the inverse of **C** . [10]

Start by forming the augmented matrix:

$$\left[\begin{array}{ccc|ccc} -1 & 1 & 0 & 1 & 0 & 0 \\ -2 & 1 & 0 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

There are many different choices for doing the row reduction, some involving 6 or more operations. Here's a choice which involves only 3 row operations:

$R_1 \to R_1 - R_2$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ -2 & 1 & 0 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 + 2R_1$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_3 \to R_3 - 2R_1$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & -2 & 2 & 1 \end{array}\right]$$

Therefore:
$$\mathbf{C}^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & - \\ -2 & 2 & 1 \end{bmatrix}$$

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(b) Based on the elementary row operations that you used to answer part (a), express the matrix **C** as a product of elementary matrices.

[10]

$R_1 \to R_1 - R_2$: $\mathbf{E}_1 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$: $\mathbf{E}_1^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_2 \to R_2 + 2R_1$: $\mathbf{E}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$: $\mathbf{E}_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_3 \to R_3 - 2R_1$: $\mathbf{E}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$: $\mathbf{E}_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

Now, $\mathbf{I} = \mathbf{E}_3\mathbf{E}_2\mathbf{E}_1\mathbf{C}$, and therefore: $\mathbf{C} = \mathbf{E}_1^{-1}\mathbf{E}_2^{-1}\mathbf{E}_3^{-1}$:

$$\mathbf{C} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

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