Math 232, Elementary Linear Algebra

J. Hebron, Spring 2000

Mid-Term Examination #2

Wednesday, March 1st, 2000 Time: 50 minutes

Student ID Number	Name (Please underline your family name)	
	Signature	

Instructions:

- Please fill-in the above information in ink.
- Do not open this exam until told to do so.
- No books, no notes, no calculators, no cell phones.
- Please sign the bottom of every page (in case your exam becomes unstapled)

Question #:	1	2	3	4	5	6	7	8	9	10	Tot
Mark:											
Out of:	3	4	6	3	9	5	7	16	12	10	75

Part A (Problem 1): True or False?

<i>Instructions:</i>	<i>Indicate whether the following</i>	statements are true or false.	No explanation
required.	, 0	,	,

requireu.	
	[mark]
1. (i) Let $\mathbf{S} = \{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2,, \vec{\mathbf{v}}_k\}$ be a set of vectors in \mathbb{R}^n . If \mathbf{S} is independent, the vector in \mathbb{R}^n can be expressed uniquely as a linear combination of vectors in \mathbf{S} .	n each [1]
(a) true (b) false	
(ii) If ${\bf H}$ is a row-echelon form of matrix ${\bf A}$, then the nonzero column vec ${\bf H}$ form a basis for the column space of ${\bf A}$.	ctors in [1]
(a) true (b) false	
(iii) The set Q of all rational numbers forms an infinite-dimensional vecto(a) true(b) false	r space. [1]
Part B (Problems 2 to 6): Short Answer Instructions: Give a short answer/solution to each of the following questions/problems. detailed explanation is required. It is assumed that any required work can be carried-outhead, so it is not necessary to show your work. Questions having very short answers are provided with an answer box – please write the answer in this box if so provided.	t in your
provinca with an answer ook - prease write the answer in this ook if so provinca.	[mark]
2. Suppose an m by n matrix A has a row space of dimension p .	
(i) What is the dimension of A's column space?	[2]
(ii) What is the dimension of A's nullspace?	[2]

(Signature)

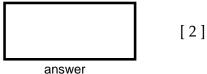
answer

3. Let **A** be the following matrix:

$$\mathbf{A} = \begin{bmatrix} a_1 & r_1a_1 & r_2a_1 & \cdots & r_{n-1}a_1 & r_na_1 \\ a_2 & r_1a_2 & r_2a_2 & \cdots & r_{n-1}a_2 & r_na_2 \\ a_3 & r_1a_3 & r_2a_3 & \cdots & r_{n-1}a_3 & r_na_3 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_{m-1} & r_1a_{m-1} & r_2a_{m-1} & \cdots & r_{n-1}a_{m-1} & r_na_{m-1} \\ a_m & r_1a_m & r_2a_m & \cdots & r_{n-1}a_m & r_na_m \end{bmatrix}$$

where $a_1 \cdots a_m$ and $r_1 \cdots r_n$ are non-zero real numbers.

(i) What is the rank of A?



(ii) Write down a basis for the row space of **A**.

[2]

(iii) Write down a basis for the column space of A.

[2]

4. If you consider differentiation to be a linear transformation **T** , what is **T**'s nullspace?

[3]

5. (i) Give an example of a vector space **V** which is not finitely generated. [You should read parts (ii) and (iii) before answering this.]

[3]

_____(Signature)

[3]

- **6.** (i) What is the determinant of the matrix **A** given in problem #3? [2]
 - answer (ii) What is the determinant of the following matrix?

 $\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & a_{1,5} \\ 0 & a_{2,2} & a_{2,3} & a_{2,4} & a_{2,5} \\ 0 & 0 & a_{3,3} & a_{3,4} & a_{3,5} \\ 0 & 0 & 0 & a_{4,4} & a_{4,5} \\ 0 & 0 & 0 & 0 & a_{5,5} \end{bmatrix}$

answer

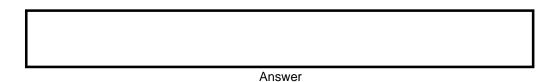
Part C (Problems 7 to 9): Show All Your Work

Instructions: Work out the following problems, showing all your work, and place the final answer in the answer box. Part marks will be awarded even if the final answer is wrong.

7. Evaluate the following determinant, where a to n and p are non-zero real numbers.

[7]:

 $\begin{vmatrix} n & l & 0 & j & c \\ b & 0 & 0 & 0 & 0 \\ h & i & a & g & p \\ k & e & 0 & 0 & 0 \\ m & f & 0 & d & 0 \\ \end{vmatrix}$

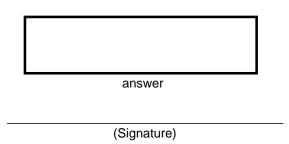


(Signature)

8. Let M_2 be the vector space of 2 by 2 matrices of real numbers. and let

$$\mathbf{B} = \left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \right)$$

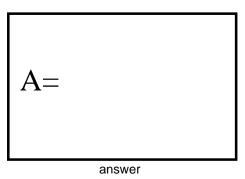
form a basis for
$$M_2$$
. What is $\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ in the basis **B**? [16]



9. Let **F** be the vector space of functions $f: \Re \to \Re$, let **V** be the subspace of **F** spanned by the basis $\mathbf{B} = (\sin(x), \cos(x), 1)$, and let **V**' be the subspace of **F** spanned by the basis $\mathbf{B'} = (e^x, e^{-x})$. Define a linear transformation $\mathbf{T}: \mathbf{V} \to \mathbf{V'}$ as follows:

$$\mathbf{T}(\sin(x)) = \frac{e^x - e^{-x}}{2}$$
$$\mathbf{T}(\cos(x)) = \frac{e^x + e^{-x}}{2}$$
$$\mathbf{T}(1) = 0$$

(i) What is the matrix representation **A** of **T** relative to **B**, **B**'? [8]



(ii) What is the kernel of T? [2]

(iii) Is the transformation invertible? Explain. [2]

Part D (Problem 10): Proof

10. Let $\bf A$ and $\bf C$ be matrices of real numbers, such that the matrix product $\bf AC$ is defined. Prove that the column space of $\bf AC$ is contained in the column space of $\bf A$.

[10]

(Signature)	