SIMON FRASER UNIVERSITY

DEPARTMENT OF MATHEMATICS AND STATISTICS

Second Midterm

MATH 232

July 15, 1998, 11:30 a.m. – 12:20 p.m.

Name: ______________________________ (please print)

family name *given name*

Signature: ______________________________

INSTRUCTIONS

1. Write your name above in block letters and sign below your name.
2. Record your answers on the answer pages found immediately below this cover sheet.
3. No calculators or other computing devices may be used.
4. This exam has 8 questions on 8 pages which follow the answer pages — please check to make sure your exam is complete.
5. Answer questions 1–6 and ONE of questions 7 and 8. If you answer both questions 7 and 8, only the one on which you score higher will count.
6. Ask for clarification if you cannot understand the question or there appears to be an error.
7. If the space provided for rough work is insufficient you may use the back of the previous page.

Answer Page 1

Question	Answer	Max	Score
1	(a) (b) (c)	2 2 2	
2	(a) (b)	3 2	
3	Yes, S is a subspace ☐ No, S is not a subspace ☐ Brief reasons:	4	
4	(a) (b)	2 4	

Subtotal	

Answer Page 2

Question	Answer	Max	Score
5	(a)	4	
	(b)	3	
6	(a)	3	
	(b)	3	
7	(a)	3	
	(b)	3	
8		6	

Subtotal	

Note: only one of questions 7 and 8 will count

Exam total	

1. Let

$$A = \begin{bmatrix} 1 & -1 & 2 & 1 & 2 \\ -4 & 2 & 1 & 1 & -3 \\ 1 & 1 & 2 & 5 & 6 \\ 0 & -1 & 1 & -1 & -1 \\ 2 & 2 & 1 & 7 & 9 \end{bmatrix}, \qquad H = \begin{bmatrix} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

It is given that H and A are row-equivalent.

[2] (a) **What is the rank of A?**

[2] (b) **Write down a basis for the row space of A.**

[2] (c) **Find a basis for the column space of A which includes $[2, -3, 6, -1, 9]$.**

ROUGH WORK

Enter your answers on the answer sheets following the cover page

2. Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $F([1,1]) = [2,-1]$ and $F([-1,1]) = [1,1]$.

[3] (a) **Find $F([1,0])$ and $F([0,1])$.**

[2] (b) **Write down the standard matrix representation of F.**

ROUGH WORK

Enter your answers on the answer sheets following the cover page

[4] **3.** Let $\mathbb{R}[x]$ denote the vector space of all polynomials over $\mathbb{R}$.

Let S denote the set of all polynomials of odd degree together with the zero polynomial.

Is S a subspace of $\mathbb{R}[x]$?

Justify your answer briefly.

ROUGH WORK

Enter your answer on the answer sheets following the cover page

4. Let V denote the space of all polynomials over $\mathbb{R}$ with degree at most 3.
Let $\mathcal{B}$ denote the ordered basis $\langle x,\, x - x^2,\, 1 + x^2,\, x^3 \rangle$ of V.
Let $\mathcal{B}'$ denote the standard basis of $\mathbb{R}^4$.
Let $F : V \to \mathbb{R}^4$ be the linear transformation defined by

$$F(a_0 + a_1 x + a_2 x^2 + a_3 x^3) = [a_3, a_2, a_1, a_0] \ .$$

[2] (a) **Find the coordinate vector $1_{\mathcal{B}}$ of 1 with respect to the ordered basis $\mathcal{B}$.**

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[4] (b) **Write down the matrix representing F with respect to $\mathcal{B}$, $\mathcal{B}'$.**

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[4] **5.** (a) Let $\boldsymbol{a} = (a_1, a_2)$, $\boldsymbol{b} = (b_1, b_2)$, $\boldsymbol{c} = (c_1, c_2)$ be points in $\mathbb{R}^2$.

Write a formula for the area of the triangle with vertices $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ in terms of the determinants

$$\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}, \quad \begin{vmatrix} c_1 & c_2 \\ a_1 & a_2 \end{vmatrix}, \quad \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}.$$

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[3] (b) Let $\boldsymbol{a} = (a_1, a_2, a_3)$, $\boldsymbol{b} = (b_1, b_2, b_3)$, $\boldsymbol{c} = (c_1, c_2, c_3)$ be points in $\mathbb{R}^3$.

What is the geometrical interpretation of the equation $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$ **?**

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[3] **6.** (a) The row operations

$$R_1 \leftrightarrow R_4,\ R_1 \to (1/3)R_1,\ R_2 \to R_2 - 3R_1,\ R_5 \to R_5 + R_1,$$
$$R_2 \to (-1/2)R_2,\ R_3 \to R_3 - 2R_2,\ R_4 \to R_4 + R_2$$

convert the 5×5 matrix A to the 5×5 identity matrix.

Evaluate det(A)**.**

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[3] (b) **Evaluate the determinant:**

$$\begin{vmatrix} 3 & 5 & 3 & 2 \\ 1 & 6 & 7 & 0 \\ 2 & -7 & 0 & 0 \\ -2 & 4 & 0 & 0 \end{vmatrix}$$

ROUGH WORK

Enter your answer on the answer sheets following the cover page

7. Let A be an $n \times n$ matrix over $\mathbb{R}$.

[3] (a) State a necessary and sufficient condition (other than the definition) for A to be diagonalizable over $\mathbb{R}$.

[3] (b) Use the criterion from your answer to part (a) to explain why the following matrix is not diagonalizable over $\mathbb{R}$:

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 2 & 0 & 1 \end{bmatrix}.$$

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[6] **8.** It is given that $\pm i$ and 2 are the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

Find an invertible matrix $C \in \mathbb{C}^{3\times 3}$ such that $C^{-1}AC$ is a diagonal matrix.

ROUGH WORK

Enter your answer on the answer sheets following the cover page