SIMON FRASER UNIVERSITY

DEPARTMENT OF MATHEMATICS AND STATISTICS

Sample Second Midterm

MATH 232

July, 1998, 55 minutes

Name: ______________________________ (please print)

family name *given name*

Signature: ______________________________

INSTRUCTIONS

1. Write your name above in block letters and sign below your name.
2. Record your answers on the answer pages found immediately below this cover sheet.
3. No calculators or other computing devices may be used.
4. This exam has 9 questions on 7 pages which follow the answer pages — please check to make sure your exam is complete.
5. Ask for clarification if you cannot understand the question or there appears to be an error.
6. If the space provided for rough work is insufficient you may use the back of the previous page.

Answer Page 1

Question	Answer	Max	Score
1		4	
2		3	
3		3	
4	Yes, S is a subspace ☐ No, S is not a subspace ☐ Brief reasons:	4	
5	(a) (b)	2 3	

Subtotal	

Answer Page 2

Question	Answer	Max	Score
6	(a) (b)	2 3	
7	(a) (b)	3 3	
8		5	
9		5	

Subtotal	

Exam total	

[4] **1. Find a basis for the subspace of $\mathbb{R}^4$ spanned by the vectors**

$$[-2, 1, 1, 3],\ [3, -2, 2, -1],\ [1, -1, 3, 2],\ [-1, 0, 4, 5]\,.$$

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[3] **2.** Let $F : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that $T([1, 3]) = [2, 1, -1]$ and $T([-2, 1]) = [0, 1, 1]$.

Find $T([1, 1])$.

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[3] 3. **Write down three axioms about scalar multiplication which are true in any vector space.**

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[4] 4. Let V denote the vector space $\mathbb{R}^{2\times 2}$ of 2×2 matrices over $\mathbb{R}$ with the usual addition and scalar multiplication.
Let S denote the set

$$\left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in V : a + b + c + d = 0 \right\}.$$

Is S a subspace of V?

Justify your answer briefly.

ROUGH WORK

Enter your answer on the answer sheets following the cover page

5. Let V denote the space of all polynomials over $\mathbb{R}$ with degree at most 3.
Let $\mathcal{B}$ denote the ordered basis $\langle 1+x,\, -1+x,\, x^2+x,\, -x^2+x^3\rangle$ of V.
Let $\mathcal{B}'$ denote the standard basis of $\mathbb{R}^2$.
Let $F : V \to \mathbb{R}^2$ be the linear transformation defined by

$$F(a_0 + a_1x + a_2x^2 + a_3x^3) = [a_3 - a_1, a_0 + a_2]\ .$$

[2] (a) **Find the coordinate vector $(1+x^3)_{\mathcal{B}}$ of $1+x^3$ with respect to $\mathcal{B}$.**

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[3] (b) **Write down the matrix representing F with respect to $\mathcal{B}$, $\mathcal{B}'$.**

ROUGH WORK

Enter your answer on the answer sheets following the cover page

6. Let $\boldsymbol{a} = (a_1, a_2)$, $\boldsymbol{b} = (b_1, b_2)$, $\boldsymbol{c} = (c_1, c_2)$ be points in the cartesian plane and O denote the origin.

[2] (a) **Write down a formula which expresses the area of the parallelogram $O\,\boldsymbol{a}\,\boldsymbol{a}+\boldsymbol{b}\,\boldsymbol{b}$ in terms of a certain determinant.**

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[3] (b) **Write down a formula for the area of the triangle $\boldsymbol{a}\,\boldsymbol{b}\,\boldsymbol{c}$ in terms of $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$.**

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[3] **7.** (a) **Evaluate the determinant:**

$$\begin{vmatrix} 0 & 2 & 0 & 0 \\ 0 & 3 & -2 & 4 \\ 1 & -2 & 1 & 5 \\ 0 & 6 & 2 & 3 \end{vmatrix}$$

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[3] (b) **Evaluate the determinant:**

$$\begin{vmatrix} 1 & 5 & 3 & 2 \\ 1 & 6 & 7 & 4 \\ 2 & 10 & 8 & 5 \\ -2 & -10 & -6 & 6 \end{vmatrix}$$

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[5] **8.** It is given that the eigenvalues of the matrix

$$A = \begin{bmatrix} i & 1 & i \\ 1 & i & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

are 0, $2+i$, and $(-1)+i$.

Find an invertible matrix $C \in \mathbb{C}^{3\times 3}$ such that $C^{-1}AC$ is a diagonal matrix.

ROUGH WORK

Enter your answer on the answer sheets following the cover page

[5] **9.** Let A denote the matrix

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

Find the eigenvalues and the corresponding eigenspaces of A.

ROUGH WORK

Enter your answer on the answer sheets following the cover page

$\overline{40}$