SIMON FRASER UNIVERSITY

DEPARTMENT OF MATHEMATICS AND STATISTICS

Second Midterm

MATH 232

March 3, 1999, 11:30 a.m. – 12:20 p.m.

Name: ______________________________ (please print)

family name *given name*

Signature: ______________________________

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Write your name above in block letters and sign below your name. Write your family name and student number in the boxes on the inside of the back cover page.
3. For each question write your final answer in the box provided.
4. No calculators or other computing devices may be used.
5. This exam has 8 questions on 7 pages — please check to make sure your exam is complete.
6. If the space provided for rough work is insufficient you may use the back of the previous page.

[3] **1.** Let $F : \mathbb{R}^2 \to \mathbb{R}^4$ be a linear transformation which satisfies

$$F([-1,3]) = [-4,6,2,0], \qquad F([2,-1]) = [3,-2,1,5]$$

Compute the standard matrix representation of F.

ANSWER

ROUGH WORK

[6] **2.** On the separate sheet circulated with the exam you will find the definition of a *vector space over* $\mathbb{R}$.

Let V be a vector space over $\mathbb{R}$.

From the axioms listed on the sheet, prove that, for all vectors $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ in V,

$$\boldsymbol{b} + \boldsymbol{a} = \boldsymbol{c} + \boldsymbol{a} \text{ implies } \boldsymbol{b} = \boldsymbol{c}\,.$$

ANSWER

[3] **3.** (a) Let V be a vector space over $\mathbb{R}$. Let W be a subset of V.

State necessary and sufficient conditions for W to be a subspace of V.

ANSWER

[4] (b) Let V be the vector space ${}^{\mathbb{R}}\mathbb{R}$ of all functions from $\mathbb{R}$ into $\mathbb{R}$. Let W denote the set

$$\left\{f \in {}^{\mathbb{R}}\mathbb{R} : (\forall x,\, y \in \mathbb{R})\,[xy > 0 \text{ implies } f(x) = f(y)]\right\} .$$

Decide whether W is a subspace of V and justify your answer.

ANSWER

[2] **4.** (a) Let V be a vector space over $\mathbb{R}$, and $\boldsymbol{v}$ be a vector in V.
Let $\mathcal{B} = \langle \boldsymbol{b}_1, \ldots, \boldsymbol{b}_n \rangle$ be an ordered basis of V.

Define the coordinate vector $\boldsymbol{v}_{\mathcal{B}}$ of $\boldsymbol{v}$ with respect to $\mathcal{B}$.

ANSWER

[3] (b) Let $\mathcal{B}$ denote the ordered basis $\langle [-1,1,2], [1,-1,0], [0,1,0] \rangle$ for $\mathbb{R}^3$.

Compute the coordinate vector of $[1,0,0]$ with respect to $\mathcal{B}$.

ANSWER

ROUGH WORK

5. Let V be the subspace of $\mathbb{R}[x]$ consisting of all polynomials of degree at most 1, and W be the subspace of $\mathbb{R}[x]$ consisting of all polynomials of degree at most 2.

Let $\mathcal{B}$, $\mathcal{C}$ denote the ordered bases

$$\langle x,\, 1\rangle, \quad \langle x^2 + x + 1,\, x + 1, 1\rangle$$

for V, W respectively.

Let $T : V \to W$ denote the linear transformation defined by $T(p) = (x + 1)p$.

[5] (a) **Compute the matrix which represents T with respect to $\mathcal{B}$, $\mathcal{C}$.**

ANSWER

[2] (b) **Is there a linear transformation $T' : W \to V$ such that $T' \circ T$ is the identity on V? Justify your answer.**

ANSWER

ROUGH WORK

(continue on back of page 3 if you need more room)

[2] **6.** (a) Let $\boldsymbol{b}$, $\boldsymbol{c}$ be vectors in $\mathbb{R}^3$.

Explain the relationship of the vector $b \times c$ to the vectors b, c in geometrical terms.

ANSWER

[2] (b) **Find the area of the triangle whose vertices are the points $(1, 1, 3)$, $(0, 1, 0)$, $(1, 1, 0)$ in $\mathbb{R}^3$.**

ANSWER

ROUGH WORK

[2] **7.** (a) **Evaluate the determinant**

$$\begin{vmatrix} a+1 & a+4 & a+7 \\ a+2 & a+5 & a+8 \\ a+4 & a+6 & a+9 \end{vmatrix}$$

ANSWER

ROUGH WORK

---

[2] (b) Let A be an $n \times n$ matrix.

Define the matrix adj(A) called the adjoint of A.

State the most important property of the adjoint of A.

ANSWER

Definition

Most important property

[2] **8.** (a) **Express the following complex number in the form $a + ib$, where $a, b \in \mathbb{R}$ and $i^2 = -1$**

$$\frac{(2-i)(1+i)}{1+2i}.$$

ANSWER

[2] (b) **Compute $r, \theta \in \mathbb{R}$, such that $r > 0$, $0 \le \theta < 2\pi$, and**

$$2 - i = r\, e^{i\theta}.$$

ANSWER

$r =$

$\theta =$

ROUGH WORK

Student number

Family name

DO NOT WRITE BELOW THIS LINE

Question	Maximum	Score
1	3	
2	6	
3	7	
4	5	
5	7	
6	4	
7	4	
8	4	
Total	40	