SIMON FRASER UNIVERSITY

DEPARTMENT OF MATHEMATICS AND STATISTICS

Second Midterm

MATH 232

November 17, 1999, 11:30 a.m. – 12:20 p.m.

Name: ______________________________ (please print)

family name *given name*

Signature: ______________________________

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.
2. Write your name above in block letters and sign below your name. Write your family name and student number in the boxes on the inside of the back cover page.
3. For each question write your final answer in the box provided.
4. No calculators or other computing devices may be used.
5. This exam has 8 questions on 8 pages — please check to make sure your exam is complete.
6. If the space provided for rough work is insufficient you may use the back of the previous page.

1. Given are four vectors $\boldsymbol{x} = [0, 1, 1, 0]$, $\boldsymbol{y} = [1, -2, 5, -1]$, $\boldsymbol{z} = [2, 1, 2, 3]$ and $\boldsymbol{w} = \boldsymbol{x} + \boldsymbol{z} = [2, 2, 3, 3]$. It is given that the set $B = \{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\}$ is a basis for the subspace V of $\mathbb{R}^4$.

[2] (a) **Let $A \in \mathbb{R}^{4\times 4}$ and let the row vectors of A be $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{w}$. Find the rank of A. Give a reason for your answer.**

ANSWER

[3] (b) **Find a basis B' for V such that $\boldsymbol{w} \in B'$. Justify your answer.**

ANSWER

ROUGH WORK

2. Let $\boldsymbol{t} = [-5, 2]$ and $\boldsymbol{u} = [3, -1]$. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation defined by $T(\boldsymbol{t}) = [1, -3, 2]$ and $T(\boldsymbol{u}) = [-1, 2, 0]$.

[3] (a) **Find the standard matrix representation of T.**

ANSWER

[1] (b) **Use the answer to part (a) to compute $T([2, 1])$.**

ANSWER

ROUGH WORK

[5] **3.** Let F be the vector space of all functions mapping $\mathbb{R}$ into $\mathbb{R}$. We say that $f \in F$ is an *odd function* if $f(-x) = -f(x)$ for every $x \in \mathbb{R}$. Let S denote the set of all odd functions.

Decide whether S is a subspace of F. Justify your answer.

ANSWER

ROUGH WORK

4. Let P_n be the vector space of all polynomials in x, with real coefficients and of degree less than or equal to n, together with the zero polynomial. Let

$$\mathcal{B} = (x^3 + x,\ x^2 - x,\ x - 1,\ 1) \quad \text{and} \quad \mathcal{B}' = (x^2,\ x,\ 1)$$

be ordered bases for P_3 and P_2, respectively. Let the linear transformation $T : P_3 \to P_2$ be defined by $T(p) = p'$, the derivative of p with respect to x.

[4] (a) **Find the matrix representation of T relative to $\mathcal{B}$, $\mathcal{B}'$.**

ANSWER

[2] (b) **Find the coordinate vector $(x^3 - x)_{\mathcal{B}}$ of $x^3 - x$ relative to $\mathcal{B}$.**

ANSWER

ROUGH WORK

[4] **5.** Given are four points $P = (1, 1, 3)$, $Q = (2, 0, 5)$, $R = (1, 4, 1)$ and $S = (3, 2, 5)$.

Decide whether P, Q, R and S are coplanar (i.e. whether they all lie in one plane in $\mathbb{R}^3$). Give a reason for your answer.

ANSWER

ROUGH WORK

[4] **6. Evaluate the determinant**

$$\begin{vmatrix} 2 & 4 & 7 & -1 & 4 \\ 0 & -3 & 2 & -1 & 4 \\ 1 & -13 & 3 & -2 & 3 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 3 & 0 & 0 & 0 \end{vmatrix}.$$

ANSWER

ROUGH WORK

[4] **7. Let $A \in \mathbb{R}^{n \times n}$. Use the known facts about determinants to prove that A is invertible if and only if 0 is not an eigenvalue of A.**

ANSWER

ROUGH WORK

[2] **8.** (a) **Let $A \in \mathbb{R}^{n \times n}$ and assume that A is diagonalizable. Explain briefly how you can compute, for any positive integer k, the power A^k using the diagonalization of A.**

ANSWER

[6] (b) **Let**

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

Use diagonalization to find the formula for A^k, $k \geq 1$.

ANSWER

ROUGH WORK *(Continue on the back of the previous page if necessary.)*

Student number

Family name

DO NOT WRITE BELOW THIS LINE

Question	Maximum	Score
1	5	
2	4	
3	5	
4	6	
5	4	
6	4	
7	4	
8	8	
Total	40	