MATH 232 98-2 Midterm 2 Answer Key

Question	Answer	Max	Score
1	(a) 3	2	
	(b) $\{[1,0,0,1,2],\ [0,1,0,2,2],\ [0,0,1,1,1]\}$	2	
	(c) $\{[2,-3,6,-1,9],[1,-4,1,0,2],[-1,2,1,-1,2]\}$	2	
2	(a) $F([1,0]) = \left[\frac{1}{2}, -1\right]$ and $F([0,1]) = \left[\frac{3}{2}, 0\right]$	3	
	(b) $\begin{bmatrix} 1/2 & 3/2 \\ -1 & 0 \end{bmatrix}$	2	
3	Yes, S is a subspace of $\mathbb{R}[x]$ ☐ No, S is not a subspace ☒ Brief reasons: The set S is not closed under vector addition because $1+x$ and $-x$ are in S but $$(1+x)+(-x) = 1 \notin S\,.$$	4	
4	(a) $1_{\mathcal{B}} = [-1,1,1,0]$	2	
	(b) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	4	
5	(a) $\frac{1}{2}\left\| \left\{ \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} + \begin{vmatrix} c_1 & c_2 \\ a_1 & a_2 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \right\} \right\|$	4	
	(b) The points $\mathbf{0}$, $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ are coplanar in $\mathbb{R}^3$.	3	
6	(a) 6	3	
	(b) 84	3	

Midterm 2 Answer Key (continued)

Question	Answer	Max	Score
7	(a) A in $\mathbb{R}^{n\times n}$ is diagonalizable over $\mathbb{R}$ if and only the sum of the dimensions of the real eigenspaces of A is n.	3	
	(b) The given matrix has eigenvalues ± 1. Each eigenspace has dimension 1. So, by (a), matrix is not diagonalizable.	3	
8	$C = \begin{bmatrix} 1 & 1-i & 1+i \\ 0 & 1+2i & 1-2i \\ 1 & -1 & -1 \end{bmatrix}$	6	