MATH 232 Key for sample Midterm 2

Question	Answer	Max	Score
1	Many possible correct answers. For example, any set consisting of two of the four given vectors.	4	
2	$T([1,1]) = (1/7)[6,1,-5]$	3	
3	Any three of S1. $r(\boldsymbol{u}+\boldsymbol{v}) = r\boldsymbol{u} + r\boldsymbol{v}$ S2. $(r+s)\boldsymbol{u} = r\boldsymbol{u} + s\boldsymbol{u}$ S3. $r(s\boldsymbol{u}) = (rs)\boldsymbol{u}$ S4. $1\boldsymbol{u} = \boldsymbol{u}$	3	
4	Yes, S is a subspace [X] No, S is not a subspace [ ] **Brief reasons:** 1. S is not empty since $O \in S$. 2. Suppose that $A_i = \begin{bmatrix} a_i & b_i \\ c_i & d_i \end{bmatrix}$ is in S for $i = 1, 2$. Then $a_i + b_i + c_i + d_i = 0$ for $i = 1, 2$. So $(a_1 + a_2) + (b_1 + b_2) + (c_1 + c_2) + (d_1 + d_2) = 0$. Hence $A_1 + A_2$ is in S, i.e., S is closed under $+$. 3. Similarly, S is closed under scalar multiplication.	4	
5	(a) $[0,-1,1,1]$ (b) $\begin{bmatrix} -1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$	2 3	
6	(a) $\left\| \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \right\|$ (absolute value of the determinant) (b) $(1/2) \left\| \begin{vmatrix} \boldsymbol{b}-\boldsymbol{a} \\ \boldsymbol{c}-\boldsymbol{a} \end{vmatrix} \right\|$ (absolute value of the determinant)	2 3	
7	(a) -28 (b) 20	3 3	

Question	Answer	Max	Score
8	$C = \begin{bmatrix} 1 & 1+2i & 1 \\ 0 & 2+i & -1 \\ -1 & 3 & 0 \end{bmatrix}$	5	
9	The characteristic polynomial of A is $$\lvert \lambda I - A \rvert = \begin{vmatrix} \lambda-1 & -2 & -2 \\ -2 & \lambda-1 & -2 \\ -2 & -2 & \lambda-1 \end{vmatrix} = (\lambda-5)(\lambda+1)^2 .$$ So the eigenvalues are $\lambda = 5$, and $\lambda = -1$ with algebraic multiplicity 2. The eigenspace of A belonging to 5 is sp$([1,1,1])$. The eigenspace of A belonging to -1 is sp$([1,-1,0],[1,0,-1])$.	5	