Math 232, Elementary Linear Algebra

J. Hebron, Spring 2000

Mid-Term Examination #3

Wednesday, March 29th, 2000

Time: 50 minutes

Student ID Number

Name
(Please underline your family name)

Signature

Instructions:

- **Please fill-in the above information in ink.**
- **Do not open this exam until told to do so.**
- **No books, no notes, no calculators, no cell phones.**
- **Please sign the bottom of every page**
  (in case your exam becomes unstapled)

Question #:	1	2	3	4	5	6	7	8	Tot
Mark:									
Out of:	8	14	9	6	8	8	5	17	75

[mark]

1. Let $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & -3 \\ 2 & 0 & 1 \end{bmatrix}$.

(a) What is the determinant of **A**? [1]

(b) What is the adjoint of **A**? [6]

(c) Based on your answers to **(a)** and **(b)**, what is the inverse of **A**? [1]

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2. **(a)** Let $a, b \in \Re$, and $i = \sqrt{-1}$. Express $\dfrac{1}{a+ib}$ in the form $c + id$, where $c, d \in \Re$.

[3]

(b) What is Euler's formula? [2]

(c) Using Euler's formula, find the three cube roots of 8, in polar form, and plot them in the complex plane.

[6]

(d) Convert the answers to **(c)** from polar form into the form $a + ib$, where $a, b \in \Re$.

[3]

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3. Let $A = \begin{bmatrix} 0 & -1 \\ i & 1+i \end{bmatrix}$, where $i = \sqrt{-1}$.

(a) What is the characteristic polynomial of **A**? [2]

(b) What are the eigenvalues of A? [3]

(c) What are the eigenvectors corresponding the eigenvalues found in **(b)**?

[4]

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4. **(a)** Define the algebraic multiplicity of an eigenvalue. [2]

(b) Define the geometric multiplicity of an eigenvalue. [2]

(c) State a necessary and sufficient condition for diagonalizability in terms of these two concepts.
[2]

5. Let $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & -2 \\ 0 & -1 & 1 \end{bmatrix}$. Find the eigenvalues of **A** and their multiplicities (both algebraic and geometric). Is **A** diagonalizable? [8]

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6 Let $\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$. Find the eigenvalues of **B** and their multiplicities (both algebraic and geometric). Is **B** diagonalizable? [8]

7. Consider the following conjecture:

If ***A*** *is an n by n real symmetric matrix with distinct eigenvalues, then* ***A*** *is invertible.*

Prove this conjecture to be false. [5]

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8. **(a)** Find a basis for the orthogonal complement in $\Re^4$ of the space $\mathbf{W} = sp([1,-1,1,1],[1,1,1,1])$. [8]

(b) Using the Gram-Schmidt process, find an orthogonal basis for **W**. [9]

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