Math 232, Elementary Linear Algebra J. Hebron, Spring 2000 Mid-Term Examination #3

> Wednesday, March 29th, 2000 Time: 50 minutes

Student ID Number

Name (Please underline your family name)

Signature

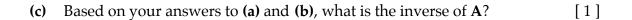
Instructions:

- Please fill-in the above information in ink.
- Do not open this exam until told to do so.
- No books, no notes, no calculators, no cell phones.
- Please sign the bottom of every page (in case your exam becomes unstapled)

Question #:	1	2	3	4	5	6	7	8	Tot
Mark:									
Out of:	8	14	9	6	8	8	5	17	75

1. Let
$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & -3 \\ 2 & 0 & 1 \end{bmatrix}$$
.
(a) What is the determinant of \mathbf{A} ? [1]

(b) What is the adjoint of A? [6]



2. (a) Let
$$a, b \in \Re$$
, and $i = \sqrt{-1}$. Express $\frac{1}{a+ib}$ in the form $c+id$, where $c, d \in \Re$.
[3]

(b) What is Euler's formula?

(c) Using Euler's formula, find the three cube roots of 8, in polar form, and plot them in the complex plane. [6]

(d) Convert the answers to (c) from polar form into the form a + ib, where $a, b \in \Re$.

[3]

[2]

3. Let
$$A = \begin{bmatrix} 0 & -1 \\ i & 1+i \end{bmatrix}$$
, where $i = \sqrt{-1}$.

(b) What are the eigenvalues of A? [3]

(c) What are the eigenvectors corresponding the eigenvalues found in (b)?

[4]

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4.	(a)	Define the algebraic multiplicity of an eigenvalue.	[2]

(b) Define the geometric multiplicity of an eigenvalue. [2]

(c) State a necessary and sufficient condition for diagonalizability in terms of these two concepts. [2]

5. Let
$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$
. Find the eigenvalues of **A** and their multiplicities (both algebraic and geometric). Is **A** diagonalizable? [8]

6 Let $\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$. Find the eigenvalues of **B** and their multiplicities (both algebraic and geometric). Is **B** diagonalizable? [8]

7. Consider the following conjecture:

If **A** is an n by n real symmetric matrix with distinct eigenvalues, then **A** is invertible.

Prove this conjecture to be false.

[5]

(b) Using the Gram-Schmidt process, find an orthogonal basis for **W**.

[9]

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