Mathematics 251–3 (Fall 1996)

Old Final Exam from Dr. Ryeburn

Final Examination **Answers** Thursday, December 5, 1996

1. Does $\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^4+y^4}$ exist? If it exists, what is its value?
Do not give an ε-δ argument, but support your conclusions with convincing reasoning.
You may use any theorems discussed in the course.

If $\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^4+y^4}$ existed then along any line $y = kx$ we would get the same value for $\lim_{x\to 0} \frac{x^4-k^4x^4}{x^4+k^4x^4}$.

But the latter limit is $\frac{1-k^4}{1+k^4}$, dependent on k.

So $\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^4+y^4}$ does not exist.

2. The distance A between $(2,1,1)$ and $(5,-5,-1)$ is exactly 7.
Use differentials to give a close approximation to the distance D between $(2.01, 1.02, 0.98)$ and $(5.03, -5.01, -1.02)$ involving indicated sums, differences, products, or quotients but **without** using square roots. Your answer should be ready for the use of a cheap calculator that can add, subtract, multiply, and divide, but you should not make any actual arithmetical calculations you would ordinarily do on such a calculator. **Think!** Don't make this a six-variable question!

$$A = \sqrt{(5-2)^2 + (-5-1)^2 + (-1-1)^2} = \sqrt{3^2 + (-6)^2 + (-2)^2} = \sqrt{49} = 7.$$

$$D = \sqrt{(5.03-2.01)^2 + (-5.01-1.02)^2 + (-1.02-0.98)^2} = \sqrt{3.02^2 + (-6.03)^2 + (-2)^2}.$$

Let $f(x,y) = \sqrt{x^2 + y^2 + 4}$, so that $A = f(3,-6)$ and $D = f(3.02, -6.03)$.
Since we want to approximate $f(3.02, -6.03)$, put $x = 3$, $y = -6$, $dx = 0.02$, and $dy = -0.03$.

$$\frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2+y^2+4}}, \text{ so } \frac{\partial f}{\partial x}(3,-6) = \frac{3}{7}.$$

$$\frac{\partial f}{\partial y} = \frac{y}{\sqrt{x^2+y^2+4}}, \text{ so } \frac{\partial f}{\partial y}(3,-6) = -\frac{6}{7}.$$

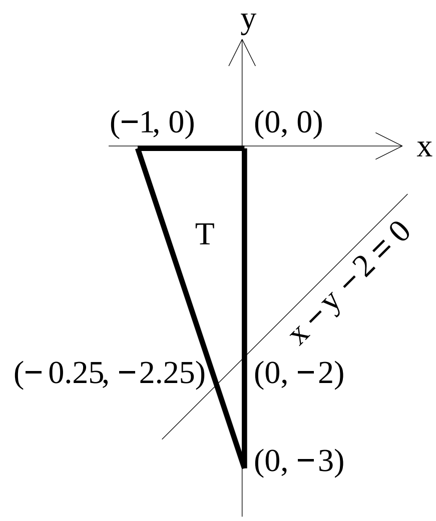
Thus $df(3,-6,0.02,-0.03) = \frac{3}{7}\ 0.02 - \frac{6}{7}\ (-0.03) = \frac{0.24}{7}$,
and $D \approx 7 + \frac{0.24}{7}$, giving the requested approximation.

My calculator approximates $7 + \frac{0.24}{7}$ as 7.034285714.

$$\sqrt{3.02^2 + (-6.03)^2 + (-2)^2} = \sqrt{9.1204 + 36.3609 + 4} = \sqrt{49.4813} \approx 7.034294563,$$

so the approximation is excellent. In fact $\dfrac{\Delta f - df}{\sqrt{dx^2 + dy^2}} \approx 0.000245421$.

3. Let T be the closed bounded triangular region in the xy-plane whose vertices are $(0,0)$, $(-1,0)$, and $(0,-3)$. Let $f(x,y) = x^2 - 2xy + y^2 - 4x + 4y + 7$. Maximize and minimize $f(x,y)$ throughout T. Make your reasoning clear!



$\frac{\partial f}{\partial x} = 2x - 2y - 4$ and $\frac{\partial f}{\partial y} = -2x + 2y + 4$ so to find the critical points we equate these to zero and solve for x and y. Unfortunately $\frac{\partial f}{\partial y} = -\frac{\partial f}{\partial x}$ so the resulting system of two equations in two unknowns has infinitely many solutions! All x and y have to do is to satisfy $x - y = 2$. There are infinitely many points in the plane where that condition holds; there are even infinitely many of them inside our triangle, along the line segment between $(-1/4, -9/4)$ and $(0, -2)$.

$\frac{\partial^2 f}{\partial x^2} = 2$, $\frac{\partial^2 f}{\partial y^2} = 2$, $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -2$, so

$\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial y \partial x}\right)^2 = 0$, telling us nothing about these points.

A bit of algebra can rescue us.

$f(x,y) = (x-y)^2 - 4(x-y) + 7 = \big((x-y) - 2\big)^2 + 3$ clearly has an absolute minimum value of 3 everywhere along the line $x - y = 2$. The minimum value, 3, of $f(x,y)$ within the triangular region T occurs everywhere along the line segment between $(-1/4, -9/4)$ and $(0, -2)$.

To find the maximum we must look along the boundary of T.

On the segment between $(-1, 0)$ and $(0, 0)$, we have $f(x,0) = x^2 - 4x + 7 = (x-2)^2 + 3$ so the maximum value along this segment is obtained by taking x as far as possible from 2, and is $f(-1,0) = 12$.

On the segment between $(0, 0)$ and $(0, -3)$, we have $f(0, y) = y^2 + 4y + 7 = (y+2)^2 + 3$ so the maximum value along this segment is obtained by taking y as far as possible from -2, and is $f(0, 0) = 7$.

On the segment between $(-1, 0)$ and $(0, -3)$, where $y = -3x - 3$,

$f(x,y) = \big((x-y) - 2\big)^2 + 3 = \big(x - (-3x-3) - 2\big)^2 + 3 = (4x+1)^2 + 3$

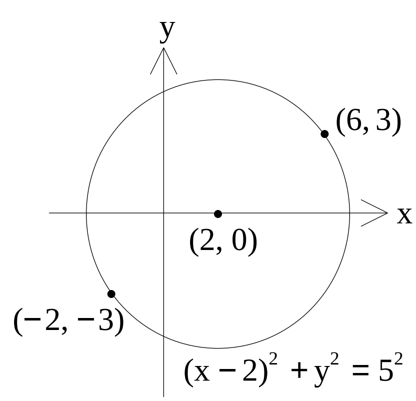
so the maximum value along this segment is obtained by taking x as far as possible from $-1/4$, and is $f(-1,0) = 12$.

The maximum value of $f(x,y)$ within the triangular region T is $f(-1,0) = 12$.

4. Use the method of Lagrange multipliers to find the absolute maximum and minimum values of the function $f(x,y) = x^2 + 4x + y^2 + 6y$ subject to the constraint $x^2 - 4x + y^2 = 21$. (The constraint condition defines a circle — a closed, bounded set.)

Note: There are many other ways to answer this question. No credit will be given unless the method of Lagrange multipliers is used.

The constraint condition can be written as $x^2 - 4x + 4 + y^2 = 25$, or
$(x-2)^2 + y^2 = 5^2$, a circle with centre $(2, 0)$ and radius 5.
Since $f(x,y) = x^2 + 4x + y^2 + 6y$ is continuous, it must have absolute maximum and minimum values somewhere on this circle.
Let $g(x, y) = x^2 - 4x + y^2$, so that our constraint is $g(x, y) = 21$.
$g(x, y) = (2x-4)\mathbf{i} + 2y\,\mathbf{j} = 2(x-2)\mathbf{i} + 2y\,\mathbf{j} \neq \mathbf{0}$ on the circle
$(x-2)^2 + y^2 = 5^2$, so we check $f(x, y) = \lambda \quad g(x,y)$.
$f(x, y) = (2x+4)\mathbf{i} + (2y+6)\,\mathbf{j} = \lambda\big((2x-4)\mathbf{i} + 2y\,\mathbf{j}\big)$.
We must solve $2x + 4 = \lambda(2x - 4)$ and $2y + 6 = \lambda(2y)$,
subject to the condition $x^2 - 4x + y^2 = 21$.
Multiplying the first equation by $2y$ and the second by $2x - 4$,
$(2x+4)(2y) = \lambda(2x-4)(2y) = (2y+6)(2x-4)$.
Eliminating the middle expression, $(2x+4)(2y) = (2y+6)(2x-4)$.
This simplifies to $8y = 12x - 8y - 24$, so $y = 0.75(x-2)$.
Substituting this into the constraint equation $(x-2)^2 + y^2 = 5^2$,
we obtain $\frac{16}{9}y^2 + y^2 = 25$, so $\frac{25}{9}y^2 = 25$, $y^2 = 9$, and $y = \pm 3$.
Since $x = 2 + \frac{4}{3}y$, our points are $(-2, -3)$ and $(6,3)$.
$f(-2,-3) = -13$ gives the minimum value and $f(6, 3) = 87$ gives the maximum value.



Actually the question can be answered (but not for credit!) geometrically.
$f(x,y) = x^2 + 4x + y^2 + 6y =$
$= (x+2)^2 + (y+3)^2 - 13$ is 13 less than the square of the distance from the point $(-2, -3)$ to the point (x, y).
This will be minimized by getting as close as possible to $(-2, -3)$ and will be maximized by getting as far as possible from $(-2, -3)$.
The point $(-2, -3)$ however is actually on our constraint circle $(x-2)^2 + y^2 = 5^2$, so we minimize $f(x,y)$ by being there.
We maximize $f(x,y)$ by being at the diametrically opposite point $(6,3)$.

5. If $f(x,y) = 2x^2 + 16xy - y^3 + 32y^2 + 300y$, find and classify all critical points of the function $f(x,y)$.

$\frac{\partial f}{\partial x} = 4x + 16y = 4(x + 4y)$ and $\frac{\partial f}{\partial y} = 16x - 3y^2 + 64y + 300$.

Equating $\frac{\partial f}{\partial x}$ to zero, $x = -4y$.

Substituting this into $\frac{\partial f}{\partial y}$ and equating the result to zero,

$-3y^2 + 300 = 0$, so $y^2 = 100$ and $y = \pm 10$.

The critical points are $(40, -10)$ and $(-40, 10)$.

$\frac{\partial^2 f}{\partial x^2} = 4$, $\frac{\partial^2 f}{\partial y^2} = -6y + 64$, and $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = 16$, so

$D = \frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 = -24y$.

$D(40, -10) = 240 > 0$, $\frac{\partial^2 f}{\partial x^2}(40, -10) = 4 > 0$, and $\frac{\partial^2 f}{\partial y^2}(40, -10) = 124 > 0$,

so $f(40, -10) = -2000$ is a local minimum value.

$D(-40, 10) = -240 < 0$, so there is a saddle point at $(-40, 10)$.

Algebraists can write $f(x,y) = 2(x + 4y)^2 + (20 - y)(y + 10)^2 - 2000$.

$2(x + 4y)^2$ cannot ever be any smaller than its zero value at $(40, -10)$.

Near $(40, -10)$ the factor $(20 - y)$ stays positive so the term

$(20 - y)(y + 10)^2$ cannot be any smaller than its zero value at $(40, -10)$

unless we move far enough away to make $y > 20$.

So the remaining term -2000 provides the local minimum value there.

Likewise $f(x,y) = 2(x + 4y)^2 - (20 + y)(y - 10)^2 + 2000$.

Near $(-40, 10)$ the factor $(20 + y)$ stays positive.

If we move away from $(-40, 10)$ in either direction along the line

$x + 4y = 0$ the first term $2(x + 4y)^2$ stays at zero but the second term

$-(20 + y)(y - 10)^2$ becomes more negative as we recede, for a while.

(It will not do this forever; it will become positive if $y < -20$.)

If we move away from $(-40, 10)$ in either direction along the line

$y = 10$ the second term $-(20 + y)(y - 10)^2$ stays at zero but the first term

$2(x + 4y)^2$ becomes more positive as we recede.

The function value at this saddle point is the remaining term 2000.

6. (a) In what direction does the function $f(x,y) = 2x^2 + 16xy - y^3 + 32y^2 + 300y$ of Question 5 increase most rapidly at the point $(0,10)$?

$$f(x,\ y) = (4x + 16y)\,\mathbf{i} + (16x - 3y^2 + 64y + 300)\,\mathbf{j} \ \text{ so}$$
$$f(0,10) = 160\,\mathbf{i} + 640\,\mathbf{j} = 160(\mathbf{i} + 4\,\mathbf{j}).$$

A unit vector in this direction is $\mathbf{u} = \frac{1}{\sqrt{17}}\,\mathbf{i} + \frac{4}{\sqrt{17}}\,\mathbf{j}$.

This is the direction in which $f(x,y)$ increases most rapidly at $(0,10)$.

(b) How rapidly does $f(x,y) = 2x^2 + 16xy - y^3 + 32y^2 + 300y$ increase in its direction of most rapid increase, at $(0,10)$?

$$|\ f(0,10)| = |160\,\mathbf{i} + 640\,\mathbf{j}| = |160(\mathbf{i} + 4\,\mathbf{j})| = 160\sqrt{1^2 + 4^2} = 160\sqrt{17}$$

is the rate of increase of $f(x,y)$ in the direction of most rapid increase, at the point $(0,10)$.

(c) How rapidly does $f(x,y) = 2x^2 + 16xy - y^3 + 32y^2 + 300y$ increase in the direction towards $(-3,14)$ at $(0,10)$?

The direction in question is that of the vector $(-3\,\mathbf{i} + 14\,\mathbf{j}) - 10\,\mathbf{j} = -3\mathbf{i} + 4\mathbf{j}$.

A unit vector in this direction is $\mathbf{v} = -\frac{3}{5}\,\mathbf{i} + \frac{4}{5}\,\mathbf{j}$.

$D_{\mathbf{v}}f(0,10) = \ f(0,10) \cdot \mathbf{v} = (160\mathbf{i} + 640\mathbf{j}) \cdot \left(-\frac{3}{5}\,\mathbf{i} + \frac{4}{5}\,\mathbf{j}\right) = 416$ gives the rate of increase of $f(x,y)$ in the direction towards $(-3,14)$ at $(0,10)$.

7. I want to make a box, with bottom but no top, in the shape of a rectangular parallelepiped. The box is to have volume 375m^3. The material used for the bottom costs \$12 per square metre and the material used for the four vertical faces costs \$2 per square metre. What dimensions give the cheapest box?

(You should find only one critical point; you need not verify that it provides a minimum.)

Let the box have width x, length y, and height z, each measured in metres. Then $xyz = 375$ so $z = \frac{375}{xy}$ and the cost of the material is

$C(x, y) = 12 \;\; xy + 2 \;\; 2 \;\; xz + 2 \;\; 2 \;\; yz = 12xy + \frac{1500}{y} + \frac{1500}{x}$.

$\frac{\partial C}{\partial x} = 12y - \frac{1500}{x^2}$ and $\frac{\partial C}{\partial y} = 12x - \frac{1500}{y^2}$.

Equating these partial derivatives to zero, $x^2y = 125 = y^2x$.

Consequently $(xy)^3 = x^3y^3 = 125^2 = 25^3$, so $xy = 25$

and finally $x = 5 = y$.

The box should have a square base 5 metres on a side, and should be 15 metres high.

It is clear that very high, or very wide, or very long boxes will be very expensive, so this unique critical point must give an absolute minimum.

Note that the base of our cheapest box will then cost \$300, and each vertical face will cost \$150 so that each pair of parallel vertical faces will together also cost \$300. We are being dimensionally fair in our allocation of money.

One could alternatively eliminate y in favour of x and z (or x in favour of y and z), but the algebra would be uglier.

Another way to answer this question is to use Lagrange multipliers.

We wish to minimize $K(x, y, z) = 12xy + 4xz + 4yz$ subject to the constraint $G(x, y, z) = xyz = 375$.

First note that $G(x,y, z) = yz\,\mathbf{i} + xz\,\mathbf{j} + xy\,\mathbf{k} \neq \mathbf{0}$ if $xyz = 375$.

$K(x, y, z) = (12y + 4z)\,\mathbf{i} + (12x + 4z)\,\mathbf{j} + (4x + 4y)\,\mathbf{k}$.

To have $K(x, y, z) = \lambda \;\; G(x, y, z)$ and $G(x, y, z) = 375$ we solve

$12y + 4z = \lambda yz$, $12x + 4z = \lambda xz$, $4x + 4y = \lambda xy$, $xyz = 375$.

Multiplying the first equation by x, the second by y, and the third by z and then comparing, we see that $12xy + 4xz = 12xy + 4yz = 4xz + 4yz$.

Since none of x, y, or z can be zero (their product is 375), it follows that $x = y$ and $z = 3x = 3y$.

Substituting into $xyz = 375$, $3x^3 = 375$ so $x = y = 5$ and $z = 15$.

Again the box height should be 15 metres and its square base should be 5 metres on a side.

8. Find the surface area of the portion of the paraboloid $z = 20 - x^2 - y^2$ between the planes $z = 4$ and $z = 11$.

The paraboloid meets the plane $z = 4$ where $x^2 + y^2 = 16$ and $z = 4$.
The paraboloid meets the plane $z = 11$ where $x^2 + y^2 = 9$ and $z = 11$.

$\frac{\partial z}{\partial x} = -2x$ and $\frac{\partial z}{\partial y} = -2y$ so $\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} = \sqrt{1 + 4x^2 + 4y^2} = \left(1 + 4r^2\right)^{1/2}$.

The surface area is $S = \int_0^{2\pi} \int_3^4 \left(1 + 4r^2\right)^{1/2} r\,dr\,d\theta = \int_0^{2\pi} \frac{1}{12}\left(1 + 4r^2\right)^{3/2} \Big]_3^4 d\theta =$

$= \int_0^{2\pi} \frac{65\sqrt{65} - 37\sqrt{37}}{12}\,d\theta = \frac{65\sqrt{65} - 37\sqrt{37}}{12}\,\theta\Big]_0^{2\pi} = \frac{65\sqrt{65} - 37\sqrt{37}}{6}\,\pi.$

9. Evaluate the integral $\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \left(x^2 + y^2\right)^{100} \, dy \, dx.$

The region of integration is a circular disk of radius 4 with centre the origin.

$$\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \left(x^2 + y^2\right)^{100} \, dy \, dx = \int_{0}^{2\pi}\int_{0}^{4}\left(r^2\right)^{100} r \, dr \, d\theta = \int_{0}^{2\pi}\int_{0}^{4} r^{201} \, dr \, d\theta =$$

$$= \int_{0}^{2\pi} \left.\frac{r^{202}}{202}\right]_0^4 d\theta = \int_{0}^{2\pi} \frac{4^{202}}{202} \, d\theta = \left.\frac{4^{202}}{202}\,\theta\right]_0^{2\pi} = \frac{4^{202}}{101}\,\pi.$$

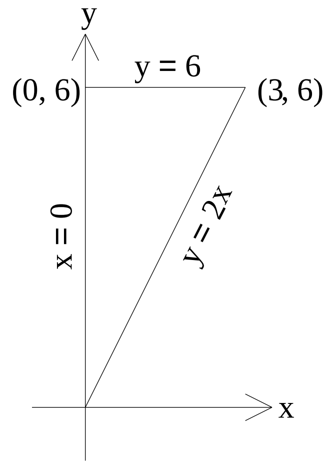
10. Evaluate the integral $\int_0^3 \int_{2x}^6 xy\cos\left(y^4\right) dy\, dx$.

Interchanging the order of integration,

$$\int_0^3 \int_{2x}^6 xy\cos\left(y^4\right) dy\, dx = \int_0^6 \int_0^{y/2} xy\cos\left(y^4\right) dx\, dy =$$

$$= \int_0^6 \frac{1}{2} x^2 y\cos\left(y^4\right)\Big]_0^{y/2} dy = \int_0^6 \frac{1}{8} y^3 \cos\left(y^4\right) dy =$$

$$= \frac{1}{32}\sin\left(y^4\right)\Big]_0^6 = \frac{1}{32}\sin 1296.$$



11. Let C be the closed curve consisting of the line segment from $(0,0,0)$ to $(1,1,5)$, followed by the line segment from $(1,1,5)$ to $(0,1,5)$, followed by the portion of the parabola $\mathbf{r}(t) = (1-t)\,\mathbf{j} + (5-5t^2)\,\mathbf{k}$ from $(0,1,5)$ to $(0,0,0)$.

Evaluate the line integral $\int_C e^{yz}\,dx + xze^{yz}\,dy + xye^{yz}\,dz$.

On the segment C_1 from $(0,0,0)$ to $(1,1,5)$ we can write
$x = t, \quad y = t, \quad z = 5t, \quad 0 \le t \le 1.$

$$\int_{C_1} e^{yz}\,dx + xze^{yz}\,dy + xye^{yz}\,dz = \int_0^1 \left[e^{5t^2}\ \ 1 + 5t^2 e^{5t^2}\ \ 1 + t^2 e^{5t^2}\ \ 5 \right] dt =$$

$$= \int_0^1 \left(10t^2 + 1\right) e^{5t^2}\,dt = te^{5t^2}\Big]_0^1 = e^5 .$$

On the segment C_2 from $(1,1,5)$ to $(0,1,5)$ we can write
$x = 1 - t, \quad y = 1, \quad z = 5, \quad 0 \le t \le 1.$

$$\int_{C_2} e^{yz}\,dx + xze^{yz}\,dy + xye^{yz}\,dz = \int_0^1 \left[e^5\ \ (-1) + 5\ \ (1-t)e^5\ \ 0 + 1\ \ (1-t)e^5\ \ 0 \right] dt =$$

$$= -e^5 t\Big]_0^1 = -e^5 .$$

On the parabola portion C_3 from $(0,1,5)$ to $(0,0,0)$ we can write
$x = 0, \quad y = 1 - t, \quad z = 5 - 5t^2, \quad 0 \le t \le 1.$

$$\int_{C_3} e^{yz}\,dx + xze^{yz}\,dy + xye^{yz}\,dz =$$

$$= \int_0^1 \left[e^{(1-t)(5-5t^2)}\ \ 0 + 0\ \ e^{(1-t)(5-5t^2)}\ \ (-1) + 0\ \ e^{(1-t)(5-5t^2)}\ \ (-10t) \right] dt = 0.$$

Summing the contributions over C_1, C_2, and C_3,

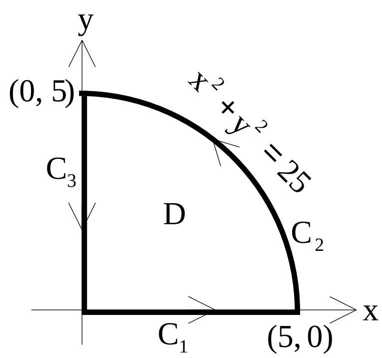
$$\int_C e^{yz}\,dx + xze^{yz}\,dy + xye^{yz}\,dz = 0.$$

This is the hard way to answer the question.
Instead, observe that $e^{yz}\,dx + xze^{yz}\,dy + xye^{yz}\,dz = d(xe^{yz})$.
Since C begins and ends at $(0,0,0)$ we have

$$\int_C e^{yz}\,dx + xze^{yz}\,dy + xye^{yz}\,dz = xe^{yz}\Big]_{(0,0,0)}^{(0,0,0)} = 0.$$

12. Let C be the closed curve consisting of the line segment from $(0,0)$ to $(5,0)$, followed by the quarter of the circle $x^2 + y^2 = 25$ from $(5,0)$ to $(0,5)$, followed by the line segment from $(0,5)$ to $(0,0)$.

Evaluate the line integral $\int_C \left(2xy^2 + y\right) dx + \left(2x^2y - x\right) dy$.



On the segment C_1 from $(0, 0)$ to $(5,0)$ we can write $x = t$, $y = 0$, $0 \le t \le 5$.

$$\int_{C_1} \left(2xy^2 + y\right)dx + \left(2x^2 y - x\right)dy =$$

$$= \int_0^5 \left(0\ \ 1 - t\ \ 0\right) dt = \int_0^5 0 dt = 0.$$

On the arc C_2 from $(5,0)$ to $(0, 5)$ we can write $x = 5\cos t$, $y = 5\sin t$, $0 \le t \le \pi/2$.

$$\int_{C_2} \left(2xy^2 + y\right)dx + \left(2x^2 y - x\right)dy =$$

$$= \int_0^{\pi/2} \left[\left(250\cos t\sin^2 t + 5\sin t\right)\ \ (-5\sin t) + \left(250\cos^2 t\sin t - 5\cos t\right)\ \ (5\cos t)\right] dt =$$

$$= \int_0^{\pi/2} \left[-1250\cos t\sin^3 t - 25\sin^2 t + 1250\cos^3 t\sin t - 25\cos^2 t\right] dt =$$

$$= \int_0^{\pi/2} \left[1250\cos t\sin t\left(\cos^2 t - \sin^2 t\right) - 25\left(\cos^2 t + \sin^2 t\right)\right] dt =$$

$$= \int_0^{\pi/2} \left[625\sin(2t)\cos(2t) - 25\right] dt = \left(156.25\sin^2(2t) - 25t\right)\Big]_0^{\pi/2} = -12.5\pi.$$

On the segment C_3 from $(0, 5)$ to $(0, 0)$ we can write $x = 0$, $y = 5 - t$, $0 \le t \le 5$.

$$\int_{C_3} \left(2xy^2 + y\right)dx + \left(2x^2 y - x\right)dy = \int_0^5 \left[(5 - t)\ \ 0 + 0\ \ (-1)\right] dt = \int_0^5 0 dt = 0.$$

Summing the contributions over C_1, C_2, and C_3,

$$\int_C \left(2xy^2 + y\right)dx + \left(2x^2 y - x\right)dy = -12.5\pi.$$

This is the hard way to answer the question.
Using Green’s Theorem instead, let D be the sector inside C.
Since $\frac{\partial}{\partial x}\left(2x^2 y - x\right) - \frac{\partial}{\partial y}\left(2xy^2 + y\right) = (4xy - 1) - (4xy + 1) = -2$,

$$\int_C \left(2xy^2 + y\right)dx + \left(2x^2 y - x\right)dy = \iint_D (-2)dA = \int_0^{\pi/2}\int_0^5 (-2) r\,dr\,d\theta =$$

$$= \int_0^{\pi/2} \left(-r^2\right)\Big]_0^5 d\theta = \int_0^{\pi/2} (-25) d\theta = (-25\theta)\Big]_0^{\pi/2} = -12.5\pi.$$