Mathematics 251–3

Old Mid-Term Exam from Dr. Ryeburn

Second Mid-Term Exam Monday, November 4, 1996

1. The upper nappe of an elliptical cone is defined by $z = \sqrt{x^2 + 4y^2}$.
The point $(3, 2, 5)$ is on this surface. Find an equation for the tangent plane at $(3, 2, 5)$ on the surface $z = \sqrt{x^2 + 4y^2}$. The arithmetic is easy and must be done completely.

2. (a) Use the differential to approximate the value of z on the elliptical cone nappe $z = \sqrt{x^2 + 4y^2}$ (the surface of Question 1) at the point where $x = 3.01$ and $y = 1.98$. The arithmetic is easy and must be done completely.
Calculators may not be used on this examination.

(b) Use the differential to approximate the value of z on the elliptical cone nappe $z = \sqrt{x^2 + 4y^2}$ (the surface of Question 1) at the point where $x = 3.03$ and $y = 2.02$. The arithmetic is easy and must be done completely.
Calculators may not be used on this examination.

(c) A dishonest student uses a calculator anyway and finds that $\sqrt{(3.03)^2 + 4\ (2.02)^2} = 5.050000000$. His calculator displays 10 significant figures, and he is surprised that the last 7 of them are zeros. Those aren't the only zeros that he will see associated with this examination!

The student later discusses the question with other students who followed instructions and used the differential to answer part (b). He is amazed that his answer was the same as theirs. Explain why the answer obtained using the differential is the same as that obtained by using a calculator. (What is so special about the values $x = 3.03$ and $y = 2.02$?)

3. If $f(x,y) = x^2 + 2xy + 2y^3 + 4y^2 + 10$, find and classify all critical points of $f(x,y)$. Show your work! If your method is correct but your algebra is missing and was incorrect, we will think your method was incorrect.

4. Consider the function $f(x,y) = x + 16y + 2$. The branch of the hyperbola $xy = 4, x > 0, y > 0$ although closed is not a bounded set, so the theorem guaranteeing the existence of a maximum and a minimum for a continuous function on a closed bounded set does not apply.

(a) Explain briefly, using words and not calculus, why $f(x,y)$ does **not** have a constrained maximum value along the hyperbola branch $xy = 4, x > 0, y > 0$.

(b) The function $f(x,y)$ **does** have a constrained minimum value along the hyperbola branch $xy = 4, x > 0, y > 0$. Find it, using the method of Lagrange multipliers. No credit will be given if any other method is used.

5. Evaluate the double integral $\int_0^1 \int_{5x}^5 e^{-y^2}\, dy\, dx$.