ODESSA - *a software environment for Analyzing Step-by-Step Solutions to Ordinary Differential Equations*

J.H. Verner

Formulation of Adams multistep and Runge–Kutta single-step formulas for numerically solving initial value problems has improved. In 1966, J.C. Butcher [1] proposed a more inclusive formulation as A-B methods in a paper which formed the basis of the first intensive study of convergence by this author. In 1980, Burrage and Butcher [2] modified the A-B form to more useful General Linear Methods (GLMs) - a form in which both traditional and new types of methods can be studied and derived. In contrast to deriving *explicit* Runge–Kutta methods [3,4], solution of GLM order conditions (see [5]) is more challenging. To facilitate their study and derivation, an algorithm first dictated by Butcher on Jan. 1, 1970, to verify the order of an 11-stage explicit Runge–Kutta method has been modified and coded for GLMs. This MAPLE code includes algebraic and numerical order estimates, graphing of stability regions, and leads to the derivation of GLMs. While its development continues, the code yields new questions in addition to the desired easier study of GLMs.

References

[1] J. C. Butcher, On the convergence of numerical solutions to ordinary differential equations, Math. Comp. **20** (1966), 1–10.

[2] K. Burrage and J. C. Butcher: Non-linear stability of a general class of differential equation methods. BIT **20** (1980), 185–203.

[3] J. H. Verner, Explicit Runge–Kutta methods with estimates of the local truncation error, SIAM J. Numer. Anal. **15** (1978), 772–790.

[4] J. H. Verner, High-order explicit Runge–Kutta pairs with low stage order, Appl Numer. Math. **22** (1996), 345–357.

[5] Anne Kværnø and J. H. Verner, Subquadrature expansions for TSRK methods, Numer. Algor. **59** (2012), 487–504