

Explicit Runge–Kutta Pairs with Lower Stage-Order *

J.H. Verner

Dedicated to John C. Butcher in
celebration of his eightieth birthday

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Abstract Explicit Runge–Kutta *pairs* of methods of successive orders of accuracy provide effective algorithms for approximating solutions to nonstiff initial value problems. For each explicit RK *method* of order of accuracy p , there is a minimum number s_p of derivative evaluations required for each step propagating the numerical solution. For $p \leq 8$, Butcher has established exact values of s_p , and for $p > 8$, his work establishes lower bounds; otherwise, upper bounds are established by various published methods. Recently, Khashin has derived some new methods *numerically*, and shown that the known upper bound on s_9 for methods of order $p = 9$ can be reduced from 15 to 13. His results motivate this attempt to identify parametrically *exact representations* for coefficients of such *methods*. New *pairs* of methods of orders 5,6 and 6,7 are characterized in terms of several arbitrary parameters. This approach, modified from an earlier one, increases the known spectrum of types of RK *pairs* and their derivations, may lead to the derivation of new RK pairs of higher-order, and possibly to other types of explicit algorithms within the class of general linear methods.

Keywords Runge–Kutta pairs · Order conditions · local error estimation · stage-order

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J.H. Verner

Department of Mathematics, Simon Fraser University, Burnaby, BC, V5A 1S6 Canada

URL: <http://www.math.sfu.ca/~jverner>

E-mail: jverner@pims.math.ca

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