Chaotic Pattern Dynamics on Sun-melted Snow

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Suncups

- spontaneous pattern formation on snow surface
- caused by differential trapping of light beneath the surface
- reduce albedo (diffuse reflection) of the snow and increase melt rate
- how do they evolve in time? how fast do they move?

Linear Stability (How the Pattern Forms)

dependence of light concentration $c$ on surface height $h$:  
- curvature augmented trapping (destabilising):  
- light diffusion (stabilising):

$$c_1 \sim \nabla^2 h$$

$$c_2 \sim \nabla \cdot J - \nabla \cdot (\nabla c_0) \sim \nabla^2 h$$

height recession:

$$\Delta h = (c_1 - c_2) - \nabla^2 h$$

in Fourier space:

$$\Delta h = (k^2 - k^4) \hat{h}$$

Dimensionless Nonlinear Evolution Equation

$$\partial_t \psi = - \nabla^2 \psi - \nabla \cdot (\nabla \psi) - 2 \sin(\psi) \nabla^2 \psi - \nabla \cdot (\psi \nabla \psi)$$

$$h = h_0 \exp(-\frac{t}{t_0})$$

$$x = x_0 \chi$$  \quad  $0 \in [\pi/2, \pi]$  

Numerical Method

- spectral Exponential Time Differencing with Runge-Kutta 4 (ETDRK4) [2]
- integrate in two space dimensions to simulate snow surface
- adaptive timestep restricted to powers of two: save ETDRK4 coefficients from varying time discretisation levels
- implemented in a combination of C and Fortran.

Finding $x_0$, $h_0$, and $t_0$

- use calibrated images of trenches cut in the snow to construct cross sections
- compare PSD to numerical simulations to determine $x_0$
- compare re-formation rate of flattened areas to numerical simulations to determine $t_0$ and $h_0$

Finding $\theta$

- observe snow surface with a webcam over a period of 50 days
- track position of individual suncups as the snow melts
- compare RMS displacement to minima in scaled numerical simulations
- displacement rate increases with $\theta$ so we find a value for $\theta$ that best fits the data

Conclusion

- we can find all parameters necessary to match the model to the observation
- both nonlinear terms is necessary to match the timescale of the chaotic dynamics
- $\theta$ parameter gives us the extra degree of freedom needed

References

[1] Tom Tiedje et. al., Radiation Transport Model for Ablation Hollows on Snowfields, J. Geophys. Res. 111 (206), F02015