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The Riemann Hypothesis: A Resource for the Afficionado and Virtuoso Alike

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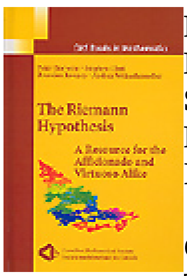

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The Riemann Hypothesis has received much attention lately, partly because of its inclusion as one of the Clay Millennium Problems and partly because, after the proof of Fermat's Last Theorem, it has become the most famous unsolved problem in mathematics. This book attempts to bridge the gap between the "popular" books (by Derbyshire, Rockmore, du Sautoy, Sabbagh, Havil…) and technical monographs (by Edwards, Titchmarsh, Ivić, Patterson…) and the even more technical research literature. Despite the subtitle, "virtuosi" will find little of interest here, unless they are historically minded. For "afficionados," this book may be useful, but only if one is prepared to overlook its eccentricities.

The Riemann Hypothesis presents itself as fundamentally a collection of well-known papers related to the Riemann Hypothesis, with a long introduction to set the stage. In fact, it includes several papers that I would describe as being on the distribution of primes but independent of the RH. *Four* papers giving elementary proofs of the Prime Number Theorem are included; of course, the whole point of such elementary proofs is that do not use properties of the zeta function. Also included are an early paper by Chebyshev on the distribution of the primes and the paper of Agrawal, Kayal, and Saxena on their polynomial-time algorithm for primality testing.

The papers more directly relevant to RH presented here include four expository accounts, Riemann's original paper (both a photographic reproduction of the original and an English translation), the proofs of the Prime Number Theorem by both Hadamard and de la Vallée Poussin, plus several others. Two papers on the pair correlation of zeros and its relation to random matrices are included as well. One of the expository papers is particularly interesting: "On Some Reasons for Doubting the Riemann Hypothesis," by Alexksandar Ivić.

Given how widely the net is cast, a few famous papers seem to be missing. For example, Euler's paper on the functional equation of the zeta function (a personal favorite of mine) is not here. At the other extreme, some account of the Weil conjectures and their proof by Deligne should have been included. One could, in fact, have made the case for several papers mentioned in the first few chapters, such as those on zero-free regions and more papers that demonstrate how RH is *used* to prove other results.

All of the papers are photographically reproduced from their original journal publication. Since many of these journals used larger pages than this book, these reproductions are often quite hard to read: the type is small (the champion here is a paper by Brian Conrey reproduced from the *Notices of the AMS*) and the images are sometimes a bit fuzzy (e.g, in the long article by de la Vallée Poussin). The photographs of Riemann's original handwritten memoir are gray and muddy, almost illegible.

Each paper has a short introduction explaining why it was included and giving a bit of biographical information about the author. In no case are we told where and when the paper was originally published. (In some cases, one can deduce this from headers or footers in the photographed pages.) There are two sections at the end labeled "References"; I am not quite sure why. One can find the bibliographic data for some of the reproduced papers in one or another of these lists, but many aren't given there either.

Preceding the papers are a few chapters giving a quick sketch of the theory. Some of these are useful, others are disappointing. The sections on computing the zeta function and on the various extensions of RH are nicely done, if a little compact. On the other hand, the chapter discussing "Equivalent Statements" missed the chance to distinguish between equivalences that are trivial and those that are deep. A deep equivalence between two statements actually tells us something that simple restatements do not.

The most disappointing of these chapters is chapter 8, "Failed Attempts at Proof." A thorough account of what has been tried without success would actually be helpful for "afficionados," maybe even for experts. But what we get is just a sort of list: Stieltjes thought he had proved something, but it turned out to be wrong; Rademacher announced a disproof, but no one except Siegel ever saw it; Louis de Branges claims to have a proof, but we're not even going to tell you what his approach is, only that Conrey and Li claim it can't work. Of course, it is potentially humiliating to display someone's mistakes, but just noting that they have happened is not helpful. In fact, this is made clear by the anecdote told by Lars Hörmander included at the head of this chapter: one must see an actual failed attempt to learn from it.

This may be a useful resource for small libraries where access to the articles included might be problematic, and for those who might like to have copies of the papers in their personal library. In general, however, it seems like an opportunity missed.

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