Math 303, Fall 2011, Lecture 2

① The empty set

So far we don't have any specific sets from our axioms. Let us start with the simplest one

Axiom of the empty set There is a set, written $\emptyset$, which contains no elements

Note ①

②

② Building sets part 1 : pairs

We started by writing out sets explicitly eg $\{1,2,3\}$ but we don't yet have axioms to do that.

Here is a start

Axiom of pairing (or unordered pairs)

For any two sets A and B, there is a set C with $A \in C$ and $B \in C$ and nothing else

write this as $C = \{A, B\}$

eg

eg

eg

Note ① $\{A, B\} =$ $($ $)$

② $\{A, A\} =$

lets check that $\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\{\{\emptyset\}\}\}, \ldots$ are all distinct.

Use the same ideas to show that all sets made from $\emptyset$ and the pairing axiom are distinct.

(2) Building sets part 2: unions

other natural thing to do is

Note (1)

(2) Compare this to the pairing axiom

3 Now we can make sets with more than two elements

But we want to do more than this

Axiom of Unions

Let $\mathcal{C}$ be a set of sets. Then there is a set which contains all elements which belong to at least one set from $\mathcal{C}$, and nothing else

Note ①

②

eg let $A_n = \{1, 2, \ldots, n\}$ for any positive n

let $\mathcal{C} = \{A_1, A_2, \ldots\}$

Properties and special case of unions

1. What is $\bigcup \emptyset$

2. What is $\bigcup \{A\}$

3. What is $\bigcup \{A, B\}$

(4) What is $A \cup \emptyset$?

(5) What is $A \cup A$?

Also Note (1)
(2)

(3) Next time

Power sets, and things we can construct with our axioms so far

$\boxed{\text{Please read sections 5 and 6 of Halmos}}$