## SOLUTIONS TO THE PRACTICE PROBLEMS FOR THE MIDTERM

MATH 303, FALL 2011

Here are some practice questions on the material since assignment 4.
(1)

| $A$ | $B$ | $A \wedge B$ | $\sim(A \wedge B)$ | $A \vee B$ | whole thing |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $F$ | $F$ | $F$ | $T$ | $F$ | $F$ |
| $F$ | $T$ | $F$ | $T$ | $T$ | $T$ |
| $T$ | $F$ | $F$ | $T$ | $T$ | $T$ |
| $T$ | $T$ | $T$ | $F$ | $T$ | $F$ |

This happens to be XOR (exclusive or).
(2) $\exists x \forall y(\sim(y \in x))$.
(3) Which of the following are well formed formulas? Which of the well formed ones are good?
(a) not well formed
(b) well formed, not good
(c) well formed, not good
(d) well formed, good
(4) Mark the free and bound variables in the well formed formulas of the previous part. Free and bound have been indicated in superscripts, match the numbers to see which are bound together.
(a) $\forall x^{\text {bound } 3} \exists y^{\text {bound } 2} \exists x^{\text {bound } 1}\left(x^{\text {bound } 1} \in y^{\text {bound } 2}\right)$
(b) $\left(\forall x^{\text {bound } 2}\left(x^{\text {bound } 2} \in y^{\text {free }}\right)\right) \vee\left(\exists y^{\text {bound } 1}\left(x^{\text {free }}=y^{\text {bound } 1}\right)\right)$
(c) $\left(y^{\text {free }} \in x^{\text {free }}\right) \wedge\left(y^{\text {free }}=x^{\text {free }}\right)$

