

# Math 303, Fall 2011, Lecture 9

## ① The axiom of choice

Motivating questions

What is  $\emptyset \times X$  for any  $X$ ?

What is  $X \times \emptyset$  for any  $X$ ?

If  $X$  and  $Y$  are nonempty can  $X \times Y = \emptyset$ ?

take  $x \in X$ ,  $y \in Y$  then  $(x, y) \in X \times Y$   
so  $X \times Y \neq \emptyset$

For infinite cartesian products we can't (always) do that with the axioms we have so we need a new axiom

## The axiom of choice

Let  $I$  be a nonempty set. Let  $\{Y_i\}_{i \in I}$  be a family of nonempty sets indexed by  $I$

$$\text{Then } \prod_{i \in I} Y_i \neq \emptyset$$

Does this seem like a reasonable axiom to take to be true?

The axiom of choice has many consequences and equivalents. Some seem completely obvious. Others seem completely unbelievable.

Let us explore some together in groups.

# Instructions

- ① Get in a group (9 groups total)
- ② Get a sheet/packet about one result related to the axiom of choice
- ③ Use Halmos, the attached copies, wikipedia etc to learn about your result
- ④ Prepare a 5 min presentation about your result  
Potential things to present (you do **not** need to do all of them)

- (a) background and definitions
- (b) the statement of your result
- (c) examples of your definitions and result
- (d) plausibility of your result and consequences for taking choice as an axiom
- (e) proof of the relation of the axiom of choice to your result

# A2 Notes

II If YOU GET INFO  
FROM WIKIPEDIA  
OR OTHER SOURCES  
THEN

**CITE IT**