Math 343, lecture 13

(1) Practicalities of Boltzmann samplers

Some important questions regarding Boltzman samplers

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② Speed

As a first attempt to answer the speed question lets forget about

Each piece

provided

oracle assumption

It is plausible in practice as

This brings up another question

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The above discussion gives

Proposition Let $\mathcal{A}$ be a combinatorial class with a specification (either iterative or recursive) in terms of the constructions we know (and some others also work)

Then under the oracle assumption the generation of $a \in \mathcal{A}$ by the Boltzmann generator of $\mathcal{A}$ takes

What about the oracle itself and sensitivity to round off errors.

③ Geometric random numbers

How to implement a geometric random number generator assuming you have a uniform random number generator:

Algorithm Geometric_rand

input λ (parameter for geometric distribution)

$p(k) = (1-\lambda)\lambda^k$

$u = \mathrm{rand}()$ (uniform random in $(0,1)$)

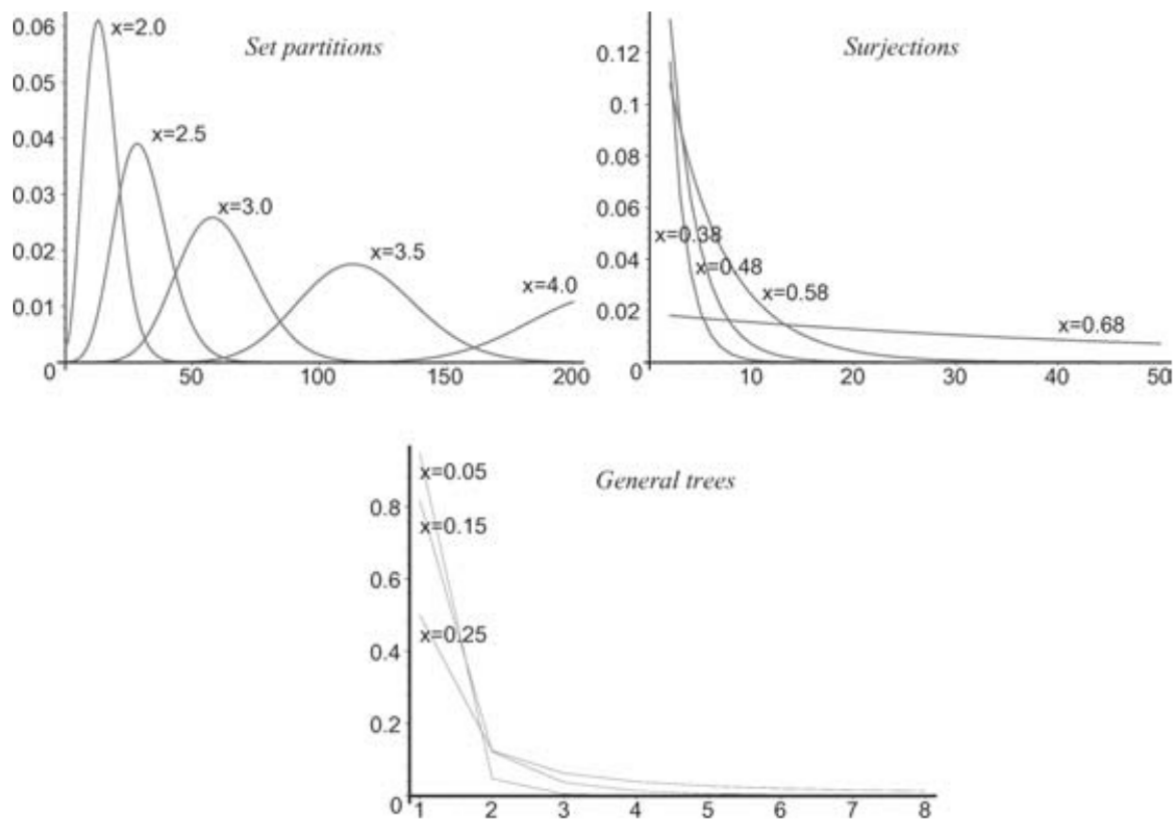
This approach will also work for other distributions just change $p(k)$.

4 Distributions and finding the optimal x

As we saw last time the expected size of the Boltzmann model with parameter x for a class $\mathcal{C}$ is

$$E_x = x\frac{A'(x)}{A(x)} \quad \text{so}$$

But



The bumpy distributions are the best for Boltzmann sampling and there are precise conditions which guarantee that they occur.

Typical examples are

What about flat distributions $\longrightarrow$ we can use a rejection strategy

With a flat distribution this strategy will succeed

Classes of words typically have flat distributions.

Finally the most troublesome ones — peaked distributions

⑤ Pointing

Consider a combinatorial class $\mathcal{C}$ built recursively out ~~of~~ $+, \times, \mathcal{E}, \mathcal{Z}, \mathrm{Seq}$

Build a new class $\mathcal{C}^\bullet$

Each $c \in \mathcal{C}$ gives

so $C^\bullet(x) =$

pointing

Using the rules for derivatives we can translate over specifications to the pointed classes

$\mathcal{Z}^\bullet$

$\mathcal{E}^\bullet$

$$(\mathcal{A}+\mathcal{B})^{\circ} =$$

$$(\mathcal{A}\times\mathcal{B})^{\circ} =$$

$$(\mathrm{Seq}\,\mathcal{A})^{\circ} =$$

eg

For the Boltzmann generator

Let's see it

⑥ Next time

Gray codes