MATH 817 ASSIGNMENT 1

DUE OCTOBER 1, 2009, IN CLASS

If your assignment must be late for any reason please notify me (by email, phone or in person) **before** the assignment is due. There will be no retroactive lates.

(1) (Isaacs problem 1.8). Let G be a group in which every nonidentity element is an involution (order 2). Show that G is abelian.

(2) (Isaacs problem 2.4). Suppose G has precisely two subgroups. Without using the Sylow theorems show that G has prime order.

(3) (Isaacs problem 2.21). Suppose $A \lhd G$ is abelian and $AH = G$ for some subgroup H. Show that $A \cap H \lhd G$.

(4) (Isaacs problem 3.4). Let π be any set of prime numbers. A finite group H is a π-*group* if all primes that divide $|H|$ lie in π. If $|G| < \infty$, then a *Hall* π-*subgroup* if G is a π-subgroup H such that $|G : H|$ is divisible by no prime in π. Let ϕ be a homomorphism defined on G.

   (a) If $H \subseteq G$ is a Hall π-subgroup, show that $\phi(H)$ is a Hall π-subgroup of $\phi(G)$.

   (b) Show that $\phi(G)$ is a π-group iff $HN = G$, where $N = \ker(\phi)$ and H is as in the previous part.

(5) Let P be a p-group. Let A be a normal subgroup of order p. Prove that A is contained in the center of P.

(6) Let G be a finite group, let $P, Q \in \mathrm{Syl}_p(G)$.

   (a) If $N_G(Q) = N_G(P)$ then $P = Q$.

   (b) $N_G(N_G(P)) = N_G(P)$.

(7) Show that a group of order 105 has a subgroup of order 35.